ELECTRIC CHARGE AND ELECTRIC FIELD

21

21.2. IDENTIFY: The charge that flows is the rate of charge flow times the duration of the time interval.

SET UP: The charge of one electron has magnitude $e = 1.60\times10^{-19}$ C.

EXECUTE: The rate of charge flow is 20,000 C/s and $t = 100\ \mu\text{s} = 1.00\times10^{-4}$ s.

$Q = (20{,}000\ \text{C/s})(1.00\times10^{-4}\ \text{s}) = 2.00$ C. The number of electrons is $n_e = \dfrac{Q}{1.60\times10^{-19}\ \text{C}} = 1.25\times10^{19}$.

EVALUATE: This is a very large amount of charge and a large number of electrons.

21.3. IDENTIFY: From your mass estimate the number of protons in your body. You have an equal number of electrons.

SET UP: Assume a body mass of 70 kg. The charge of one electron is -1.60×10^{-19} C.

EXECUTE: The mass is primarily protons and neutrons of $m = 1.67\times10^{-27}$ kg. The total number of protons and neutrons is $n_{\text{p and n}} = \dfrac{70\ \text{kg}}{1.67\times10^{-27}\ \text{kg}} = 4.2\times10^{28}$. About one-half are protons, so $n_p = 2.1\times10^{28} = n_e$. The number of electrons is about 2.1×10^{28}. The total charge of these electrons is
$Q = (-1.60\times10^{-19}\ \text{C/electron})(2.10\times10^{28}\ \text{electrons}) = -3.35\times10^{9}$ C.

EVALUATE: This is a huge amount of negative charge. But your body contains an equal number of protons and your net charge is zero. If you carry a net charge, the number of excess or missing electrons is a very small fraction of the total number of electrons in your body.

21.7. IDENTIFY: Apply Coulomb's law.

SET UP: Consider the force on one of the spheres.

(a) EXECUTE: $q_1 = q_2 = q$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = \frac{q^2}{4\pi\epsilon_0 r^2} \text{ so } q = r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = 0.150\ \text{m}\sqrt{\frac{0.220\ \text{N}}{8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 7.42\times10^{-7}\ \text{C (on each)}$$

(b) $q_2 = 4q_1$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = \frac{4q_1^2}{4\pi\epsilon_0 r^2} \text{ so } q_1 = r\sqrt{\frac{F}{4(1/4\pi\epsilon_0)}} = \tfrac{1}{2}r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = \tfrac{1}{2}(7.42\times10^{-7}\ \text{C}) = 3.71\times10^{-7}\ \text{C}.$$

And then $q_2 = 4q_1 = 1.48\times10^{-6}$ C.

EVALUATE: The force on one sphere is the same magnitude as the force on the other sphere, whether the sphere have equal charges or not.

21.8. IDENTIFY: Use the mass of a sphere and the atomic mass of aluminum to find the number of aluminum atoms in one sphere. Each atom has 13 electrons. Apply Coulomb's law and calculate the magnitude of charge $|q|$ on each sphere.

SET UP: $N_A = 6.02\times10^{23}$ atoms/mol. $|q| = n_e' e$, where n_e' is the number of electrons removed from one sphere and added to the other.

EXECUTE: **(a)** The total number of electrons on each sphere equals the number of protons.

$$n_e = n_p = (13)(N_A)\left(\frac{0.0250\ \text{kg}}{0.026982\ \text{kg/mol}}\right) = 7.25\times10^{24}\ \text{electrons}.$$

21.13. **IDENTIFY:** Apply Coulomb's law. The two forces on q_3 must have equal magnitudes and opposite directions.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: The force $\vec{F}_2$ that q_2 exerts on q_3 has magnitude $F_2 = k\dfrac{|q_2 q_3|}{r_2^2}$ and is in the +*x* direction. $\vec{F}_1$ must be in the $-x$ direction, so q_1 must be positive. $F_1 = F_2$ gives $k\dfrac{|q_1||q_3|}{r_1^2} = k\dfrac{|q_2||q_3|}{r_2^2}$.

$$|q_1| = |q_2|\left(\frac{r_1}{r_2}\right)^2 = (3.00\text{ nC})\left(\frac{2.00\text{ cm}}{4.00\text{ cm}}\right)^2 = 0.750\text{ nC}.$$

EVALUATE: The result for the magnitude of q_1 doesn't depend on the magnitude of q_2.

21.25. **IDENTIFY:** $F = |q|E$. Since the field is uniform, the force and acceleration are constant and we can use a constant acceleration equation to find the final speed.

SET UP: A proton has charge +*e* and mass 1.67×10^{-27} kg.

EXECUTE: **(a)** $F = (1.60\times10^{-19}\text{ C})(2.75\times10^{3}\text{ N/C}) = 4.40\times10^{-16}\text{ N}$

(b) $a = \dfrac{F}{m} = \dfrac{4.40\times10^{-16}\text{ N}}{1.67\times10^{-27}\text{ kg}} = 2.63\times10^{11}\text{ m/s}^2$

(c) $v_x = v_{0x} + a_x t$ gives $v = (2.63\times10^{11}\text{ m/s}^2)(1.00\times10^{-6}\text{ s}) = 2.63\times10^{5}\text{ m/s}$

EVALUATE: The acceleration is very large and the gravity force on the proton can be ignored.

21.34. **IDENTIFY:** Apply Eq.(21.7) to calculate the electric field due to each charge and add the two field vectors to find the resultant field.

SET UP: For q_1, $\hat{\boldsymbol{r}} = \hat{\boldsymbol{j}}$. For q_2, $\hat{\boldsymbol{r}} = \cos\theta\hat{\boldsymbol{i}} + \sin\theta\hat{\boldsymbol{j}}$, where θ is the angle between $\vec{E}_2$ and the +*x*-axis.

EXECUTE: **(a)** $\vec{E}_1 = \dfrac{q_1}{4\pi\epsilon_0 r_1^2}\hat{\boldsymbol{j}} = \dfrac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(-5.00\times10^{-9}\text{ C})}{(0.0400\text{ m})^2} = (-2.813\times10^{4}\text{ N/C})\hat{\boldsymbol{j}}$.

$\left|\vec{E}_2\right| = \dfrac{q_2}{4\pi\epsilon_0 r_2^2} = \dfrac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-9}\text{ C})}{(0.0300\text{ m})^2 + (0.0400\text{ m})^2} = 1.080\times10^{4}\text{ N/C}$. The angle of $\vec{E}_2$, measured from the *x*-axis, is $180° - \tan^{-1}\left(\dfrac{4.00\text{ cm}}{3.00\text{ cm}}\right) = 126.9°$ Thus

$\vec{E}_2 = (1.080\times10^{4}\text{ N/C})(\hat{\boldsymbol{i}}\cos126.9° + \hat{\boldsymbol{j}}\sin126.9°) = (-6.485\times10^{3}\text{ N/C})\hat{\boldsymbol{i}} + (8.64\times10^{3}\text{ N/C})\hat{\boldsymbol{j}}$

(b) The resultant field is $\vec{E}_1 + \vec{E}_2 = (-6.485\times10^{3}\text{ N/C})\hat{\boldsymbol{i}} + (-2.813\times10^{4}\text{ N/C} + 8.64\times10^{3}\text{ N/C})\hat{\boldsymbol{j}}$.

$$\vec{E}_1 + \vec{E}_2 = (-6.485\times10^{3}\text{ N/C})\hat{\boldsymbol{i}} - (1.95\times10^{4}\text{ N/C})\hat{\boldsymbol{j}}.$$

EVALUATE: $\vec{E}_1$ is toward q_1 since q_1 is negative. $\vec{E}_2$ is directed away from q_2, since q_2 is positive.

21.47. **IDENTIFY:** $E = k\dfrac{|q|}{r^2}$. The net field is the vector sum of the fields due to each charge.

SET UP: The electric field of a negative charge is directed toward the charge. Label the charges q_1, q_2 and q_3, as shown in Figure 21.47a. This figure also shows additional distances and angles. The electric fields at point *P* are shown in Figure 21.47b. This figure also shows the *xy* coordinates we will use and the *x* and *y* components of the fields $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$.

EXECUTE: $E_1 = E_3 = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-6}\text{ C}}{(0.100\text{ m})^2} = 4.49\times10^{6}\text{ N/C}$

$E_2 = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{2.00\times10^{-6}\text{ C}}{(0.0600\text{ m})^2} = 4.99\times10^{6}\text{ N/C}$

$E_y = E_{1y} + E_{2y} + E_{3y} = 0$ and $E_x = E_{1x} + E_{2x} + E_{3x} = E_2 + 2E_1\cos53.1° = 1.04\times10^{7}\text{ N/C}$

$E = 1.04\times10^{7}\text{ N/C}$, toward the $-2.00\ \mu\text{C}$ charge.

EVALUATE: The *x*-components of the fields of all three charges are in the same direction.

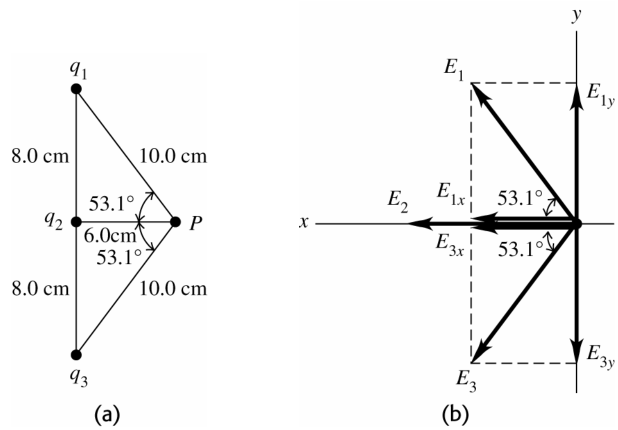


Figure 21.47

21.71. **(a) IDENTIFY:** Use Coulomb's law to calculate each force and then add them as vectors to obtain the net force. Torque is force times moment arm.

SET UP: The two forces on each charge in the dipole are shown in Figure 21.71a.

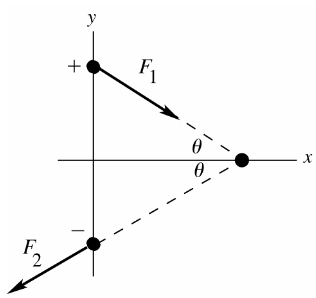


Figure 21.71a

$\sin\theta = 1.50/2.00$ so $\theta = 48.6°$

Opposite charges attract and like charges repel.

$F_x = F_{1x} + F_{2x} = 0$

EXECUTE: $F_1 = k\frac{|qq'|}{r^2} = k\frac{(5.00\times10^{-6}\text{ C})(10.0\times10^{-6}\text{ C})}{(0.0200\text{ m})^2} = 1.124\times10^3\text{ N}$

$F_{1y} = -F_1\sin\theta = -842.6\text{ N}$

$F_{2y} = -842.6\text{ N}$ so $F_y = F_{1y} + F_{2y} = -1680\text{ N}$ (in the direction from the $+5.00\text{-}\mu\text{C}$ charge toward the $-5.00\text{-}\mu\text{C}$ charge).

EVALUATE: The *x*-components cancel and the *y*-components add.

(b) SET UP: Refer to Figure 21.71b.

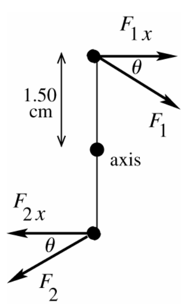


Figure 21.71b

The *y*-components have zero moment arm and therefore zero torque.

F_{1x} and F_{2x} both produce clockwise torques.

EXECUTE: $F_{1x} = F_1\cos\theta = 743.1\text{ N}$

$\tau = 2(F_{1x})(0.0150\text{ m}) = 22.3\text{ N}\cdot\text{m}$, clockwise

EVALUATE: The electric field produced by the $-10.00\mu\text{C}$ charge is not uniform so Eq. (21.15) does not apply.

21.74. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to one of the spheres.

SET UP: The free-body diagram is sketched in Figure 21.74. F_e is the repulsive Coulomb force between the spheres. For small θ, $\sin\theta \approx \tan\theta$.

EXECUTE: $\sum F_x = T\sin\theta - F_e = 0$ and $\sum F_y = T\cos\theta - mg = 0$. So $\dfrac{mg\sin\theta}{\cos\theta} = F_e = \dfrac{kq^2}{d^2}$. But $\tan\theta \approx \sin\theta = \dfrac{d}{2L}$,

so $d^3 = \dfrac{2kq^2L}{mg}$ and $d = \left(\dfrac{q^2L}{2\pi\epsilon_0 mg}\right)^{1/3}$.

EVALUATE: d increases when q increases.

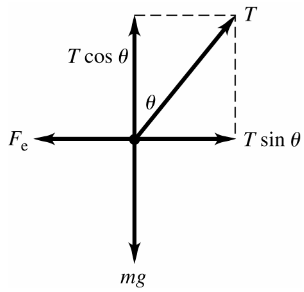


Figure 21.74

21.84. **IDENTIFY:** The electric field exerts equal and opposite forces on the two balls, causing them to swing away from each other. When the balls hang stationary, they are in equilibrium so the forces on them (electrical, gravitational, and tension in the strings) must balance.

SET UP: (a) The force on the left ball is in the direction of the electric field, so it must be positive, while the force on the right ball is opposite to the electric field, so it must be negative.

(b) Balancing horizontal and vertical forces gives $qE = T\sin\theta/2$ and $mg = T\cos\theta/2$.

EXECUTE: Solving for the angle θ gives: $\theta = 2\arctan(qE/mg)$.

(c) As $E \to \infty$, $\theta \to 2\arctan(\infty) = 2(\pi/2) = \pi = 180°$

EVALUATE: If the field were large enough, the gravitational force would not be important, so the strings would be horizontal.

21.89. **IDENTIFY:** Divide the charge distribution into infinitesimal segments of length dx. Calculate E_x and E_u due to a segment and integrate to find the total field.

SET UP: The charge dQ of a segment of length dx is $dQ = (Q/a)dx$. The distance between a segment at x and the charge q is $a + r - x$. $(1-y)^{-1} \approx 1 + y$ when $|y| << 1$.

EXECUTE: **(a)** $dE_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{dQ}{(a+r-x)^2}$ so $E_x = \dfrac{1}{4\pi\epsilon_0}\displaystyle\int_0^a \dfrac{Qdx}{a(a+r-x)^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{a}\left(\dfrac{1}{r} - \dfrac{1}{a+r}\right)$.

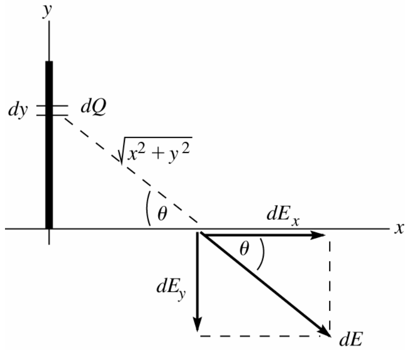
$a + r = x$, so $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{a}\left(\dfrac{1}{x-a} - \dfrac{1}{x}\right)$. $E_y = 0$.

(b) $\vec{F} = q\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{a}\left(\dfrac{1}{x-a} - \dfrac{1}{x}\right)\hat{\boldsymbol{i}}$.

EVALUATE: **(c)** For $x >> a$, $F = \dfrac{kqQ}{ax}((1-a/x)^{-1} - 1) = \dfrac{kqQ}{ax}(1 + a/x + \cdots - 1) \approx \dfrac{kqQ}{x^2} \approx \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{r^2}$. (Note that for $x >> a$, $r = x - a \approx x$.) The charge distribution looks like a point charge from far away, so the force takes the form of the force between a pair of point charges.

21.90. **IDENTIFY:** Use Eq. (21.7) to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.11. Use Eq. (21.3) to calculate $\vec{F}$.

SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.90.



$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

Figure 21.90

Slice the charge distribution up into small pieces of length *dy*. The charge *dQ* in each slice is $dQ = Q(dy/a)$. The electric field this produces at a distance *x* along the *x*-axis is *dE*. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{x^2+y^2}\right) = \frac{Q}{4\pi\epsilon_0 a}\left(\frac{dy}{x^2+y^2}\right)$

$$dE_x = dE\cos\theta = \frac{Qx}{4\pi\epsilon_0 a}\left(\frac{dy}{\left(x^2+y^2\right)^{3/2}}\right)$$

$$dE_y = -dE\sin\theta = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{ydy}{\left(x^2+y^2\right)^{3/2}}\right)$$

$$E_x = \int dE_x = -\frac{Qx}{4\pi\epsilon_0 a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{Qx}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]_0^a = \frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}$$

$$E_y = \int dEy = -\frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{ydy}{(x^2+y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^a = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+a^2}}\right)$$

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21.96. **IDENTIFY:** Divide the semicircle into infinitesimal segments. Find the electric field $d\vec{E}$ due to each segment and integrate over the semicircle to find the total electric field.

SET UP: The electric fields along the *x*-direction from the left and right halves of the semicircle cancel. The remaining *y*-component points in the negative *y*-direction. The charge per unit length of the semicircle is $\lambda = \frac{Q}{\pi a}$ and $dE = \frac{k\lambda\, dl}{a^2} = \frac{k\lambda\, d\theta}{a}$.

EXECUTE: $dE_y = dE\sin\theta = \frac{k\lambda\sin\theta\, d\theta}{a}$. Therefore, $E_y = \frac{2k\lambda}{a}\int_0^{\pi/2}\sin\theta\, d\theta = \frac{2k\lambda}{a}[-\cos\theta]_0^{\pi/2} = \frac{2k\lambda}{a} = \frac{2kQ}{\pi a^2}$, in the $-y$-direction .

EVALUATE: For a full circle of charge the electric field at the center would be zero. For a quarter-circle of charge, in the first quadrant, the electric field at the center of curvature would have nonzero *x* and *y* components. The calculation for the semicircle is particularly simple, because all the charge is the same distance from point *P*.

21.99. **IDENTIFY:** Each wire produces an electric field at *P* due to a finite wire. These fields add by vector addition.

SET UP: Each field has magnitude $\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}$. The field due to the negative wire points to the left, while the field due to the positive wire points downward, making the two fields perpendicular to each other and of equal magnitude. The net field is the vector sum of these two, which is $E_{net} = 2E_1\cos 45° = 2\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\cos 45°$. In part (b), the electrical force on an electron at *P* is *eE*.

EXECUTE: **(a)** The net field is $E_{net} = 2\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\cos 45°$.

$$E_{net} = \frac{2\left(9.00\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.50\times10^{-6}\,\text{C}\right)\cos 45°}{(0.600\text{ m})\sqrt{(0.600\text{ m})^2+(0.600\text{ m})^2}} = 6.25\times10^{4}\text{ N/C}.$$

The direction is 225° counterclockwise from an axis pointing to the right through the positive wire.
(b) $F = eE = (1.60 \times 10^{-19}\text{ C})(6.25 \times 10^{4}\text{ N/C}) = 1.00 \times 10^{-14}\text{ N}$, opposite to the direction of the electric field, since the electron has negative charge.

EVALUATE: Since the electric fields due to the two wires have equal magnitudes and are perpendicular to each other, we only have to calculate one of them in the solution.