22

GAUSS'S LAW

22.1. **(a) IDENTIFY** and **SET UP:** $\Phi_E = \int E\cos\phi\, dA,$ where ϕ is the angle between the normal to the sheet $\hat{\boldsymbol{n}}$ and the electric field $\vec{\boldsymbol{E}}$.

EXECUTE: In this problem E and $\cos\phi$ are constant over the surface so

$\Phi_E = E\cos\phi \int dA = E\cos\phi A = (14\text{ N/C})(\cos 60°)(0.250\text{ m}^2) = 1.8\text{ N}\cdot\text{m}^2\text{/C}.$

(b) EVALUATE: Φ_E is independent of the shape of the sheet as long as ϕ and E are constant at all points on the sheet.

(c) EXECUTE: (i) $\Phi_E = E\cos\phi A.$ Φ_E is largest for $\phi = 0°,$ so $\cos\phi = 1$ and $\Phi_E = EA.$

(ii) Φ_E is smallest for $\phi = 90°,$ so $\cos\phi = 0$ and $\Phi_E = 0.$

EVALUATE: Φ_E is 0 when the surface is parallel to the field so no electric field lines pass through the surface.

22.2. **IDENTIFY:** The field is uniform and the surface is flat, so use $\Phi_E = EA\cos\phi$.

SET UP: ϕ is the angle between the normal to the surface and the direction of $\vec{\boldsymbol{E}}$, so $\phi = 70°$.

EXECUTE: $\Phi_E = (75.0\text{ N/C})(0.400\text{ m})(0.600\text{ m})\cos 70° = 6.16\text{ N}\cdot\text{m}^2\text{/C}$

EVALUATE: If the field were perpendicular to the surface the flux would be $\Phi_E = EA = 18.0\text{ N}\cdot\text{m}^2\text{/C}.$ The flux in this problem is much less than this because only the component of $\vec{\boldsymbol{E}}$ perpendicular to the surface contributes to the flux.

22.3. **IDENTIFY:** The electric flux through an area is defined as the product of the component of the electric field perpendicular to the area times the area.

(a) SET UP: In this case, the electric field is perpendicular to the surface of the sphere, so $\Phi_E = EA = E(4\pi r^2)$.

EXECUTE: Substituting in the numbers gives

$$\Phi_E = (1.25\times10^6\text{ N/C})4\pi(0.150\text{ m})^2 = 3.53\times10^5\text{ N}\cdot\text{m}^2\text{/C}$$

(b) IDENTIFY: We use the electric field due to a point charge.

SET UP: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$

EXECUTE: Solving for q and substituting the numbers gives

$$q = 4\pi\epsilon_0 r^2 E = \frac{1}{9.00\times10^9\text{ N}\cdot\text{m}^2\text{/C}^2}(0.150\text{ m})^2(1.25\times10^6\text{ N/C}) = 3.13\times10^{-6}\text{ C}$$

EVALUATE: The flux would be the same no matter how large the circle, since the area is proportional to r^2 while the electric field is proportional to $1/r^2$.

22.8. **IDENTIFY:** Apply Gauss's law to each surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by each surface. Flux out of the volume is positive and flux into the enclosed volume is negative.

EXECUTE: **(a)** $\Phi_{S_1} = q_1/\epsilon_0 = (4.00\times10^{-9}\text{ C})/\epsilon_0 = 452\text{ N}\cdot\text{m}^2\text{/C}.$

(b) $\Phi_{S_2} = q_2/\epsilon_0 = (-7.80\times10^{-9}\text{ C})/\epsilon_0 = -881\text{ N}\cdot\text{m}^2\text{/C}.$

(c) $\Phi_{S_3} = (q_1 + q_2)/\epsilon_0 = ((4.00 - 7.80)\times10^{-9}\text{ C})/\epsilon_0 = -429\text{ N}\cdot\text{m}^2\text{/C}.$

(d) $\Phi_{S_4} = (q_1 + q_3)/\epsilon_0 = ((4.00 + 2.40)\times10^{-9}\text{ C})/\epsilon_0 = 723\text{ N}\cdot\text{m}^2\text{/C}.$

(e) $\Phi_{S_5} = (q_1 + q_2 + q_3)/\epsilon_0 = ((4.00 - 7.80 + 2.40)\times10^{-9}\text{ C})/\epsilon_0 = -158\text{ N}\cdot\text{m}^2\text{/C}.$

EVALUATE: **(f)** All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

22.10. **IDENTIFY:** Apply Gauss's law to the spherical surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by the sphere.

EXECUTE: **(a)** No charge enclosed so $\Phi = 0$.

(b) $\Phi = \dfrac{q_2}{\epsilon_0} = \dfrac{-6.00\times10^{-9}\text{ C}}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = -678\text{ N}\cdot\text{m}^2/\text{C}.$

(c) $\Phi = \dfrac{q_1+q_2}{\epsilon_0} = \dfrac{(4.00-6.00)\times10^{-9}\text{ C}}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = -226\text{ N}\cdot\text{m}^2/\text{C}.$

EVALUATE: Negative flux corresponds to flux directed into the enclosed volume. The net flux depends only on the net charge enclosed by the surface and is not affected by any charges outside the enclosed volume.

22.26. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that lies wholly within he conducting material.

EXECUTE: **(a)** Positive charge is attracted to the inner surface of the conductor by the charge in the cavity. Its magnitude is the same as the cavity charge: $q_{inner} = +6.00\text{ nC},$ since $E = 0$ inside a conductor and a Gaussian surface that lies wholly within the conductor must enclose zero net charge.

(b) On the outer surface the charge is a combination of the net charge on the conductor and the charge "left behind" when the $+6.00\text{ nC}$ moved to the inner surface:

$$q_{tot} = q_{inner} + q_{outer} \Rightarrow q_{outer} = q_{tot} - q_{inner} = 5.00\text{ nC} - 6.00\text{ nC} = -1.00\text{ nC}.$$

EVALUATE: The electric field outside the conductor is due to the charge on its surface.

22.44. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that is a sphere of radius *r* and that has the point charge at its center.

EXECUTE: **(a)** For $r < a$, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2},$ radially outward, since the charge enclosed is *Q*, the charge of the point charge. For $a < r < b$, $E = 0$ since these points are within the conducting material. For $r > b$, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2Q}{r^2}$, radially inward, since the total enclosed charge is –2*Q*.

(b) Since a Gaussian surface with radius *r*, for $a < r < b$, must enclose zero net charge, the total charge on the inner surface is $-Q$ and the surface charge density on inner surface is $\sigma = -\dfrac{Q}{4\pi a^2}$.

(c) Since the net charge on the shell is $-3Q$ and there is $-Q$ on the inner surface, there must be $-2Q$ on the outer surface. The surface charge density on the outer surface is $\sigma = -\dfrac{2Q}{4\pi b^2}.$

(d) The field lines and the locations of the charges are sketched in Figure 22.44a.

(e) The graph of *E* versus *r* is sketched in Figure 22.44b.

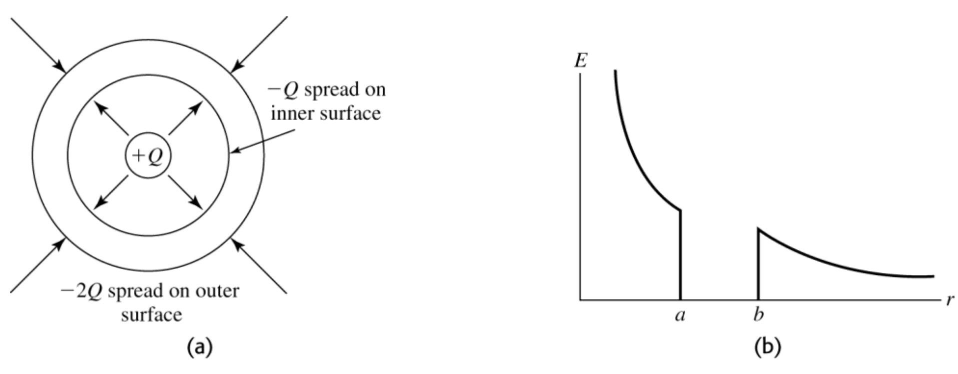

Figure 22.44

EVALUATE: For $r < a$ the electric field is due solely to the point charge *Q*. For $r > b$ the electric field is due to the charge $-2Q$ that is on the outer surface of the shell.

22.57. $\rho(r) = \rho_0(1 - r/R)$ for $r \le R$ where $\rho_0 = 3Q/\pi R^3$. $\rho(r) = 0$ for $r \ge R$

(a) IDENTIFY: The charge density varies with *r* inside the spherical volume. Divide the volume up into thin concentric shells, of radius *r* and thickness *dr*. Find the charge *dq* in each shell and integrate to find the total charge.

SET UP: The thin shell is sketched in Figure 22.57a.

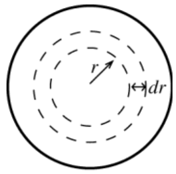

Figure 22.57a

EXECUTE: The volume of such a shell is $dV = 4\pi r^2 dr$
The charge contained within the shell is
$dq = \rho(r)dV = 4\pi r^2 \rho_0 (1 - r/R)dr$

The total charge Q in the charge distribution is obtained by integrating dq over all such shells into which the sphere can be subdivided:

$$Q = \int dq = \int_0^R 4\pi r^2 \rho_0 (1 - r/R)dr = 4\pi\rho_0 \int_0^R (r^2 - r^3/R)dr$$

$Q = 4\pi\rho_0 \left[\dfrac{r^3}{3} - \dfrac{r^4}{4R}\right]_0^R = 4\pi\rho_0 \left(\dfrac{R^3}{3} - \dfrac{R^4}{4R}\right) = 4\pi\rho_0 (R^3/12) = 4\pi(3Q/\pi R^3)(R^3/12) = Q,$ as was to be shown.

(b) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r > R$.
SET UP: The Gaussian surface is shown in Figure 22.57b.

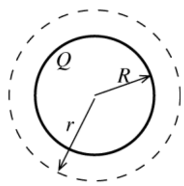

Figure 22.57b

EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$

$E(4\pi r^2) = \dfrac{Q}{\epsilon_0}$

$E = \dfrac{Q}{4\pi\epsilon_0 r^2}$; same as for point charge of charge Q.

(c) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r < R$:
SET UP: The Gaussian surface is shown in Figure 22.57c.

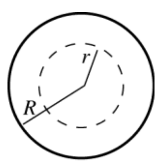

Figure 22.57c

EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$

$\Phi_E = E(4\pi r^2)$

The calculate the enclosed charge Q_{encl} use the same technique as in part (a), except integrate dq out to r rather than R. (We want the charge that is inside radius r.)

$$Q_{\text{encl}} = \int_0^r 4\pi r'^2 \rho_0 \left(1 - \frac{r'}{R}\right)dr' = 4\pi\rho_0 \int_0^r \left(r'^2 - \frac{r'^3}{R}\right)dr'$$

$$Q_{\text{encl}} = 4\pi\rho_0 \left[\frac{r'^3}{3} - \frac{r'^4}{4R}\right]_0^r = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) = 4\pi\rho_0 r^3 \left(\frac{1}{3} - \frac{r}{4R}\right)$$

$\rho_0 = \dfrac{3Q}{\pi R^3}$ so $Q_{\text{encl}} = 12Q\dfrac{r^3}{R^3}\left(\dfrac{1}{3} - \dfrac{r}{4R}\right) = Q\left(\dfrac{r^3}{R^3}\right)\left(4 - 3\dfrac{r}{R}\right).$

Thus Gauss's law gives $E(4\pi r^2) = \dfrac{Q}{\epsilon_0}\left(\dfrac{r^3}{R^3}\right)\left(4 - 3\dfrac{r}{R}\right)$

$E = \dfrac{Qr}{4\pi\epsilon_0 R^3}\left(4 - \dfrac{3r}{R}\right),\ r \le R$

(d) The graph of E versus r is sketched in Figure 22.57d.

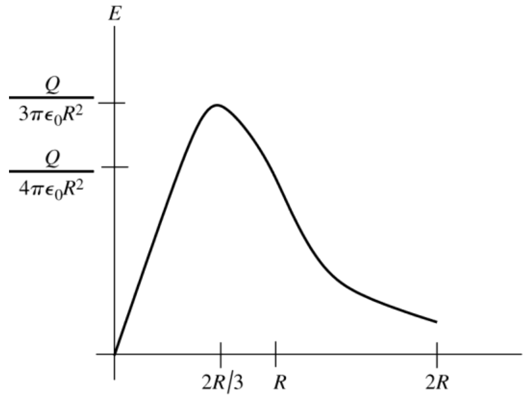

Figure 22.57d

(e) Where the electric field is a maximum, $\frac{dE}{dr}=0.$ Thus $\frac{d}{dr}\left(4r-\frac{3r^2}{R}\right)=0$ so $4-6r/R=0$ and $r=2R/3.$

At this value of r, $E=\frac{Q}{4\pi\epsilon_0 R^3}\left(\frac{2R}{3}\right)\left(4-\frac{3}{R}\frac{2R}{3}\right)=\frac{Q}{3\pi\epsilon_0 R^2}$

EVALUATE: Our expressions for $E(r)$ for $r<R$ and for $r>R$ agree at $r=R.$ The results of part (e) for the value of r where $E(r)$ is a maximum agrees with the graph in part (d).