23

ELECTRIC POTENTIAL

23.1. **IDENTIFY:** Apply Eq.(23.2) to calculate the work. The electric potential energy of a pair of point charges is given by Eq.(23.9).

SET UP: Let the initial position of q_2 be point a and the final position be point b, as shown in Figure 23.1.

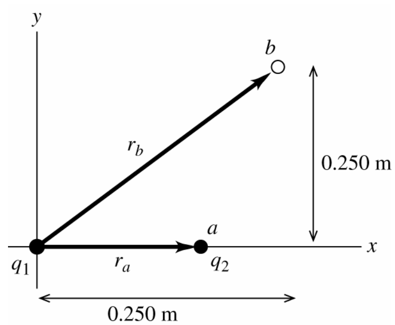


Figure 23.1

$$r_a = 0.150 \text{ m}$$

$$r_b = \sqrt{(0.250 \text{ m})^2 + (0.250 \text{ m})^2}$$

$$r_b = 0.3536 \text{ m}$$

EXECUTE: $W_{a \to b} = U_a - U_b$

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_a} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times10^{-6}\text{ C})(-4.30\times10^{-6}\text{ C})}{0.150 \text{ m}}$$

$$U_a = -0.6184 \text{ J}$$

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_b} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times10^{-6}\text{ C})(-4.30\times10^{-6}\text{ C})}{0.3536 \text{ m}}$$

$$U_b = -0.2623 \text{ J}$$

$$W_{a\to b} = U_a - U_b = -0.6184 \text{ J} - (-0.2623 \text{ J}) = -0.356 \text{ J}$$

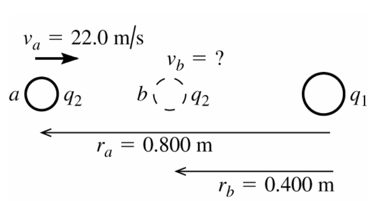
EVALUATE: The attractive force on q_2 is toward the origin, so it does negative work on q_2 when q_2 moves to larger r.

23.5. **(a) IDENTIFY:** Use conservation of energy:

$$K_a + U_a + W_{\text{other}} = K_b + U_b$$

U for the pair of point charges is given by Eq.(23.9).

SET UP:



Let point a be where q_2 is 0.800 m from q_1 and point b be where q_2 is 0.400 m from q_1, as shown in Figure 23.5a.

Figure 23.5a

EXECUTE: Only the electric force does work, so $W_{\text{other}} = 0$ and $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$.

$$K_a = \tfrac{1}{2}mv_a^2 = \tfrac{1}{2}(1.50\times10^{-3}\text{ kg})(22.0\text{ m/s})^2 = 0.3630\text{ J}$$

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_a} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\text{ C})(-7.80\times10^{-6}\text{ C})}{0.800\text{ m}} = +0.2454\text{ J}$$

$$K_b = \tfrac{1}{2}mv_b^2$$

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_b} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\text{ C})(-7.80\times10^{-6}\text{ C})}{0.400\text{ m}} = +0.4907\text{ J}$$

The conservation of energy equation then gives $K_b = K_a + (U_a - U_b)$

$$\tfrac{1}{2}mv_b^2 = +0.3630\text{ J} + (0.2454\text{ J} - 0.4907\text{ J}) = 0.1177\text{ J}$$

$$v_b = \sqrt{\frac{2(0.1177\text{ J})}{1.50\times10^{-3}\text{ kg}}} = 12.5\text{ m/s}$$

EVALUATE: The potential energy increases when the two positively charged spheres get closer together, so the kinetic energy and speed decrease.

(b) IDENTIFY: Let point c be where q_2 has its speed momentarily reduced to zero. Apply conservation of energy to points a and c: $K_a + U_a + W_{\text{other}} = K_c + U_c$.

SET UP: Points a and c are shown in Figure 23.5b.

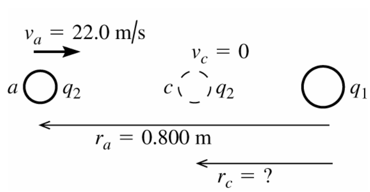


Figure 23.5b

EXECUTE: $K_a = +0.3630$ J (from part (a))

$U_a = +0.2454$ J (from part (a))

$K_c = 0$ (at distance of closest approach the speed is zero)

$$U_c = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_c}$$

Thus conservation of energy $K_a + U_a = U_c$ gives $\dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r_c} = +0.3630\text{ J} + 0.2454\text{ J} = 0.6084\text{ J}$

$$r_c = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{0.6084\text{ J}} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\text{ C})(-7.80\times10^{-6}\text{ C})}{+0.6084\text{ J}} = 0.323\text{ m.}$$

EVALUATE: $U \to \infty$ as $r \to 0$ so q_2 will stop no matter what its initial speed is.

23.8. IDENTIFY: Call the three charges 1, 2 and 3. $U = U_{12} + U_{13} + U_{23}$

SET UP: $U_{12} = U_{23} = U_{13}$ because the charges are equal and each pair of charges has the same separation, 0.500 m.

EXECUTE: $U = \dfrac{3kq^2}{0.500\text{ m}} = \dfrac{3k(1.2\times10^{-6}\text{ C})^2}{0.500\text{ m}} = 0.078\text{ J.}$

EVALUATE: When the three charges are brought in from infinity to the corners of the triangle, the repulsive electrical forces between each pair of charges do negative work and electrical potential energy is stored.

23.10. IDENTIFY: The work done on the alpha particle is equal to the difference in its potential energy when it is moved from the midpoint of the square to the midpoint of one of the sides.

SET UP: We apply the formula $W_{a\to b} = U_a - U_b$. In this case, a is the center of the square and b is the midpoint of one of the sides. Therefore $W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}}$.

There are 4 electrons, so the potential energy at the center of the square is 4 times the potential energy of a single alpha-electron pair. At the center of the square, the alpha particle is a distance $r_1 = \sqrt{50}$ nm from each electron. At the midpoint of the side, the alpha is a distance r_2 = 5.00 nm from the two nearest electrons and a distance r_2 = $\sqrt{125}$ nm from the two most distant electrons. Using the formula for the potential energy (relative to infinity) of two point charges, $U = (1/4\pi\epsilon_0)(qq_0/r)$, the total work is

$$W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}} = 4\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_1} - \left(2\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_2} + 2\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_3}\right)$$

Substituting $q_e = e$ and $q_\alpha = 2e$ and simplifying gives

$$W_{\text{center}\to\text{side}} = -4e^2\frac{1}{4\pi\epsilon_0}\left[\frac{2}{r_1} - \left(\frac{1}{r_2} + \frac{1}{r_3}\right)\right]$$

EXECUTE: Substituting the numerical values into the equation for the work gives

$$W = -4\left(1.60\times10^{-19}\text{ C}\right)^2\left[\frac{2}{\sqrt{50}\text{ m}} - \left(\frac{1}{5.00\text{ nm}} + \frac{1}{\sqrt{125}\text{ nm}}\right)\right] = 6.08\times10^{-21}\text{ J ???}$$

EVALUATE: Since the work is positive, the system has more potential energy with the alpha particle at the center of the square than it does with it at the midpoint of a side.

23.12. **IDENTIFY:** Use conservation of energy $U_a + K_a = U_b + K_b$ to find the distance of closest approach r_b. The maximum force is at the distance of closest approach, $F = k\frac{|q_1 q_2|}{r_b^2}$.

SET UP: $K_b = 0$. Initially the two protons are far apart, so $U_a = 0$. A proton has mass 1.67×10^{-27} kg and charge $q = +e = +1.60\times10^{-19}$ C.

EXECUTE: $K_a = U_b$. $2(\tfrac{1}{2}mv_a^2) = k\frac{q_1 q_2}{r_b}$. $mv_a^2 = k\frac{e^2}{r_b}$ and

$$r_b = \frac{ke^2}{mv_a^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(1.67\times10^{-27}\text{ kg})(1.00\times10^6\text{ m/s})^2} = 1.38\times10^{-13}\text{ m.}$$

$$F = k\frac{e^2}{r_b^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\text{ C})^2}{(1.38\times10^{-13}\text{ C})^2} = 0.012\text{ N.}$$

EVALUATE: The acceleration $a = F/m$ of each proton produced by this force is extremely large.

23.16. **IDENTIFY:** The work-energy theorem says $W_{a\to b} = K_b - K_a$. $\frac{W_{a\to b}}{q} = V_a - V_b$.

SET UP: Point a is the starting point and point b is the ending point. Since the field is uniform, $W_{a\to b} = Fs\cos\phi = E|q|s\cos\phi$. The field is to the left so the force on the positive charge is to the left. The particle moves to the left so $\phi = 0°$ and the work $W_{a\to b}$ is positive.

EXECUTE: **(a)** $W_{a\to b} = K_b - K_a = 1.50\times10^{-6}\text{ J} - 0 = 1.50\times10^{-6}\text{ J}$

(b) $V_a - V_b = \frac{W_{a\to b}}{q} = \frac{1.50\times10^{-6}\text{ J}}{4.20\times10^{-9}\text{ C}} = 357\text{ V}$. Point a is at higher potential than point b.

(c) $E|q|s = W_{a\to b}$, so $E = \frac{W_{a\to b}}{|q|s} = \frac{V_a - V_b}{s} = \frac{357\text{ V}}{6.00\times10^{-2}\text{ m}} = 5.95\times10^3\text{ V/m}$.

EVALUATE: A positive charge gains kinetic energy when it moves to lower potential; $V_b < V_a$.

23.19. **IDENTIFY** and **SET UP:** For a point charge $V = \frac{kq}{r}$. Solve for r.

EXECUTE: **(a)** $r = \frac{kq}{V} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.50\times10^{-11}\text{ C})}{90.0\text{ V}} = 2.50\times10^{-3}\text{ m} = 2.50\text{ mm}$

(b) $Vr = kq = \text{constant}$ so $V_1 r_1 = V_2 r_2$. $r_2 = r_1\left(\frac{V_1}{V_2}\right) = (2.50\text{ mm})\left(\frac{90.0\text{ V}}{30.0\text{ V}}\right) = 7.50\text{ mm}$.

EVALUATE: The potential of a positive charge is positive and decreases as the distance from the point charge increases.

23.22. **IDENTIFY:** For a point charge, $V = \frac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: **(a)** The positions of the two charges are shown in Figure 23.22a. $r=\sqrt{a^2+x^2}$.

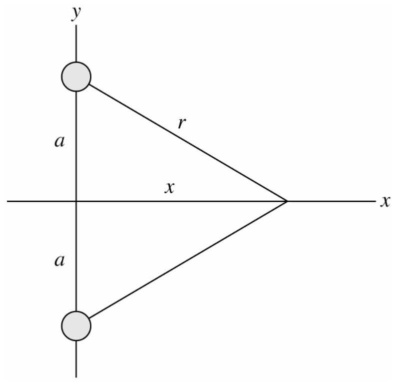


Figure 23.22a

EXECUTE: **(b)** $V_0=2\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{a}$.

(c) $V(x)=2\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}=2\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{\sqrt{a^2+x^2}}$

(d) The graph of V versus x is sketched in Figure 23.22b.

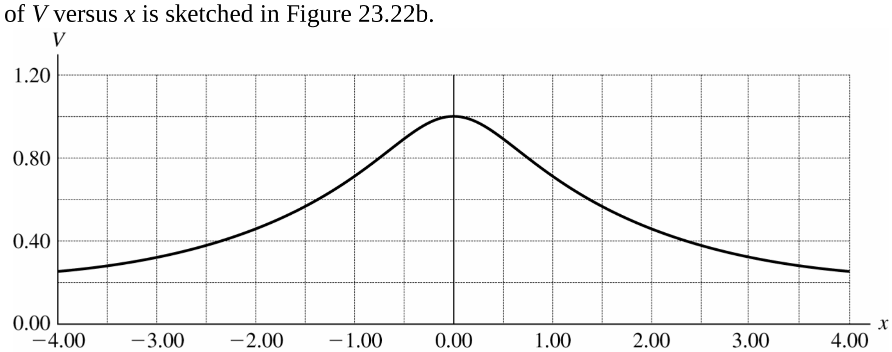


Figure 23.22b

EVALUATE: **(e)** When $x>>a$, $V=\dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{x}$, just like a point charge of charge $+2q$. At distances from the charges much greater than their separation, the two charges act like a single point charge.

23.31. **IDENTIFY** and **SET UP:** Apply conservation of energy, Eq.(23.3). Use Eq.(23.12) to express U in terms of V.

(a) EXECUTE: $K_1+qV_1=K_2+qV_2$

$q(V_1-V_2)=K_2-K_1;$ $\quad q=-1.602\times10^{-19}$ C

$K_1=\frac{1}{2}m_e v_1^2=4.099\times10^{-18}$ J; $\quad K_2=\frac{1}{2}m_e v_2^2=2.915\times10^{-17}$ J

$V_1-V_2=\dfrac{K_2-K_1}{q}=-156$ V

EVALUATE: The electron gains kinetic energy when it moves to higher potential.

(b) EXECUTE: Now $K_1=2.915\times10^{-17}$ J, $K_2=0$

$$V_1-V_2=\frac{K_2-K_1}{q}=+182\text{ V}$$

EVALUATE: The electron loses kinetic energy when it moves to lower potential.

23.33. **(a) IDENTIFY** and **SET UP:** The electric field on the ring's axis is calculated in Example 21.10. The force on the electron exerted by this field is given by Eq.(21.3).

EXECUTE: When the electron is on either side of the center of the ring, the ring exerts an attractive force directed toward the center of the ring. This restoring force produces oscillatory motion of the electron along the axis of the ring, with amplitude 30.0 cm. The force on the electron is *not* of the form $F=-kx$ so the oscillatory motion is not simple harmonic motion.

(b) IDENTIFY: Apply conservation of energy to the motion of the electron.

SET UP: $K_a + U_a = K_b + U_b$ with *a* at the initial position of the electron and *b* at the center of the ring. From Example 23.11, $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2+R^2}}$, where *R* is the radius of the ring.

EXECUTE: $x_a = 30.0$ cm, $x_b = 0$.

$K_a = 0$ (released from rest), $K_b = \frac{1}{2}mv^2$

Thus $\frac{1}{2}mv^2 = U_a - U_b$

And $U = qV = -eV$ so $v = \sqrt{\frac{2e(V_b - V_a)}{m}}$.

$$V_a = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_a^2+R^2}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\ \text{C}}{\sqrt{(0.300\ \text{m})^2+(0.150\ \text{m})^2}}$$

$$V_a = 643\ \text{V}$$

$$V_b = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_b^2+R^2}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\ \text{C}}{0.150\ \text{m}} = 1438\ \text{V}$$

$$v = \sqrt{\frac{2e(V_b - V_a)}{m}} = \sqrt{\frac{2(1.602\times10^{-19}\ \text{C})(1438\ \text{V} - 643\ \text{V})}{9.109\times10^{-31}\ \text{kg}}} = 1.67\times10^7\ \text{m/s}$$

EVALUATE: The positively charged ring attracts the negatively charged electron and accelerates it. The electron has its maximum speed at this point. When the electron moves past the center of the ring the force on it is opposite to its motion and it slows down.

23.42. IDENTIFY: The electric field is zero inside the sphere, so the potential is constant there. Thus the potential at the center must be the same as at the surface, where it is equivalent to that of a point-charge.

SET UP: At the surface, and hence also at the center of the sphere, the field is that of a point-charge, $E = Q/(4\pi\epsilon_0 R)$.

EXECUTE: **(a)** Solving for *Q* and substituting the numbers gives

$$Q = 4\pi\epsilon_0 RV = (0.125\ \text{m})(1500\ \text{V})/(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 2.08\times10^{-8}\ \text{C} = 20.8\ \text{nC}$$

(b) Since the potential is constant inside the sphere, its value at the surface must be the same as at the center, 1.50 kV.

EVALUATE: The electric field inside the sphere is zero, so the potential is constant but is not zero.

23.45. IDENTIFY: Example 23.9 shows that $V(y) = Ey$, where *y* is the distance from the negatively charged plate, whose potential is zero. The electric field between the plates is uniform and perpendicular to the plates.

SET UP: *V* increases toward the positively charged plate. $\vec{E}$ is directed from the positively charged plated toward the negatively charged plate.

EXECUTE: **(a)** $E = \frac{V}{d} = \frac{480\ \text{V}}{0.0170\ \text{m}} = 2.82\times10^4\ \text{V/m}$ and $y = \frac{V}{E}$. $V = 0$ at $y = 0$, $V = 120$ V at $y = 0.43$ cm, $V = 240$ V at $y = 0.85$ cm, $V = 360$ V at $y = 1.28$ cm and $V = 480$ V at $y = 1.70$ cm. The equipotential surfaces are sketched in Figure 23.45. The surfaces are planes parallel to the plates.

(b) The electric field lines are also shown in Figure 23.45. The field lines are perpendicular to the plates and the equipotential lines are parallel to the plates, so the electric field lines and the equipotential lines are mutually perpendicular.

EVALUATE: Only differences in potential have physical significance. Letting $V = 0$ at the negative plate is a choice we are free to make.

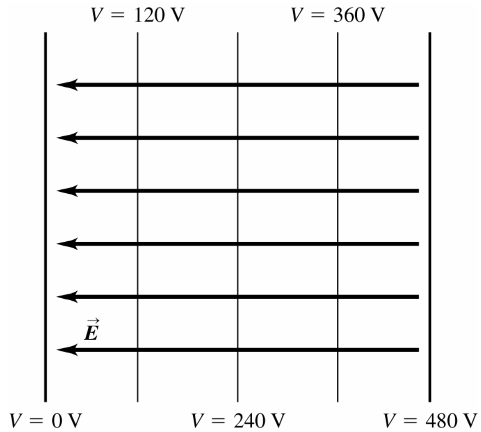


Figure 23.45

23.47 **IDENTIFY** and **SET UP:** Use Eq.(23.19) to calculate the components of $\vec{E}$.

EXECUTE: $V = Axy - Bx^2 + Cy$

(a) $E_x = -\dfrac{\partial V}{\partial x} = -Ay + 2Bx$

$Ey = -\dfrac{\partial V}{\partial y} = -Ax - C$

$E_z = -\dfrac{\partial V}{\partial z} = 0$

(b) $E = 0$ requires that $E_x = E_y = E_z = 0.$

$E_z = 0$ everywhere.

$E_y = 0$ at $x = -C/A.$

And E_x is also equal zero for this *x*, any value of *z*, and $y = 2Bx/A = (2B/A)(-C/A) = -2BC/A^2$.

EVALUATE: *V* doesn't depend on *z* so $E_z = 0$ everywhere.