24

CAPACITANCE AND DIELECTRICS

24.1. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$

SET UP: $1\ \mu\text{F} = 10^{-6}\ \text{F}$

EXECUTE: $Q = CV_{ab} = (7.28\times10^{-6}\ \text{F})(25.0\ \text{V}) = 1.82\times10^{-4}\ \text{C} = 182\ \mu\text{C}$

EVALUATE: One plate has charge $+Q$ and the other has charge $-Q$.

24.5. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, $V_{ab} = 12.0\ \text{V}$.

EXECUTE: **(a)** $Q = CV_{ab} = (10.0\times10^{-6}\ \text{F})(12.0\ \text{V}) = 1.20\times10^{-4}\ \text{C} = 120\ \mu\text{C}$

(b) When *d* is doubled *C* is halved, so *Q* is halved. $Q = 60\ \mu\text{C}$.

(c) If *r* is doubled, *A* increases by a factor of 4. *C* increases by a factor of 4 and *Q* increases by a factor of 4. $Q = 480\ \mu\text{C}$.

EVALUATE: When the plates are moved apart, less charge on the plates is required to produce the same potential difference. With the separation of the plates constant, the electric field must remain constant to produce the same potential difference. The electric field depends on the surface charge density, σ. To produce the same σ, more charge is required when the area increases.

24.14. **IDENTIFY:** The capacitors between *b* and *c* are in parallel. This combination is in series with the 15 pF capacitor.

SET UP: Let $C_1 = 15\ \text{pF}$, $C_2 = 9.0\ \text{pF}$ and $C_3 = 11\ \text{pF}$.

EXECUTE: **(a)** For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + \cdots$ so $C_{23} = C_2 + C_3 = 20\ \text{pF}$

(b) $C_1 = 15\ \text{pF}$ is in series with $C_{23} = 20\ \text{pF}$. For capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$ so $\dfrac{1}{C_{123}} = \dfrac{1}{C_1} + \dfrac{1}{C_{23}}$ and

$$C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{(15\ \text{pF})(20\ \text{pF})}{15\ \text{pF} + 20\ \text{pF}} = 8.6\ \text{pF}.$$

EVALUATE: For capacitors in parallel the equivalent capacitance is larger than any of the individual capacitors. For capacitors in series the equivalent capacitance is smaller than any of the individual capacitors.

24.15. **IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for *Q* and *V* for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

SET UP: Do parts **(a)** and **(b)** together. The capacitor network is drawn in Figure 24.15a.

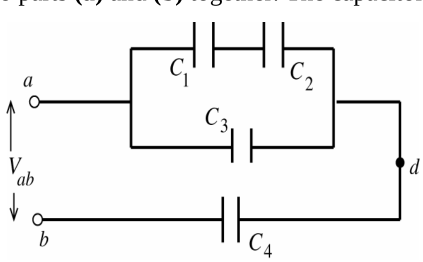
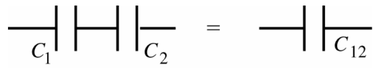


Figure 24.15a

$C_1 = C_2 = C_3 = C_4 = 400\ \mu\text{F}$
$V_{ab} = 28.0\ \text{V}$

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_1 and C_2 are in series and are equivalent to C_{12} (Figure 24.15b).

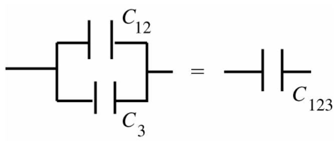


$$\frac{1}{C_{12}}=\frac{1}{C_1}+\frac{1}{C_2}$$

Figure 24.15b

$$C_{12}=\frac{C_1C_2}{C_1+C_2}=\frac{\left(4.00\times10^{-6}\text{ F}\right)\left(4.00\times10^{-6}\text{ F}\right)}{4.00\times10^{-6}\text{ F}+4.00\times10^{-6}\text{ F}}=2.00\times10^{-6}\text{ F}$$

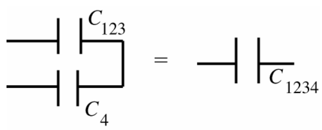
C_{12} and C_3 are in parallel and are equivalent to C_{123} (Figure 24.15c).



$$C_{123}=C_{12}+C_3$$
$$C_{123}=2.00\times10^{-6}\text{ F}+4.00\times10^{-6}\text{ F}$$
$$C_{123}=6.00\times10^{-6}\text{ F}$$

Figure 24.15c

C_{123} and C_4 are in series and are equivalent to C_{1234} (Figure 24.15d).

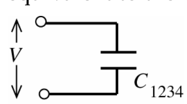


$$\frac{1}{C_{1234}}=\frac{1}{C_{123}}+\frac{1}{C_4}$$

Figure 24.15d

$$C_{1234}=\frac{C_{123}C_4}{C_{123}+C_4}=\frac{\left(6.00\times10^{-6}\text{ F}\right)\left(4.00\times10^{-6}\text{ F}\right)}{6.00\times10^{-6}\text{ F}+4.00\times10^{-6}\text{ F}}=2.40\times10^{-6}\text{ F}$$

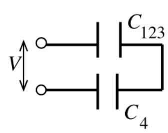
The circuit is equivalent to the circuit shown in Figure 24.15e.



$$V_{1234}=V=28.0\text{ V}$$
$$Q_{1234}=C_{1234}V=\left(2.40\times10^{-6}\text{ F}\right)\left(28.0\text{ V}\right)=67.2\ \mu\text{C}$$

Figure 24.15e

Now build back up the original circuit, step by step. C_{1234} represents C_{123} and C_4 in series (Figure 24.15f).



$$Q_{123}=Q_4-Q_{1234}=67.2\ \mu\text{C}$$
(charge same for capacitors in series)

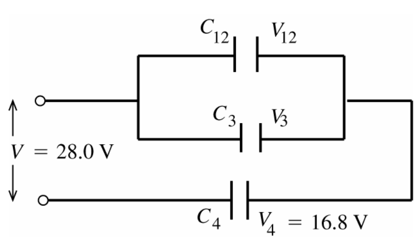
Figure 24.15f

Then $V_{123}=\dfrac{Q_{123}}{C_{123}}=\dfrac{67.2\ \mu\text{C}}{6.00\ \mu\text{F}}=11.2\text{ V}$

$V_4=\dfrac{Q_4}{C_4}=\dfrac{67.2\ \mu\text{C}}{4.00\ \mu\text{F}}=16.8\text{ V}$

Note that $V_4+V_{123}=16.8\text{ V}+11.2\text{ V}=28.0\text{ V}$, as it should.

Next consider the circuit as written in Figure 24.15g.



$$V_3=V_{12}=28.0\text{ V}-V_4$$
$$V_3=11.2\text{ V}$$
$$Q_3=C_3V_3=\left(4.00\ \mu\text{F}\right)\left(11.2\text{ V}\right)$$
$$Q_3=44.8\ \mu\text{C}$$
$$Q_{12}-C_{12}V_{12}=\left(2.00\ \mu\text{F}\right)\left(11.2\text{ V}\right)$$
$$Q_{12}=22.4\ \mu\text{C}$$

Figure 24.15g

Finally, consider the original circuit, as shown in Figure 24.15h.

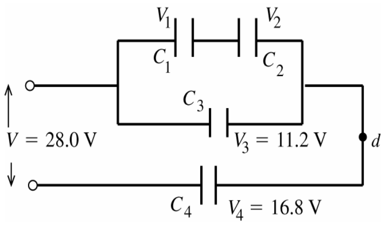


Figure 24.15h

$Q_1 = Q_2 = Q_{12} = 22.4\ \mu\text{C}$

(charge same for capacitors in series)

$$V_1 = \frac{Q_1}{C_1} = \frac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6\ \text{V}$$

$$V_2 = \frac{Q_2}{C_2} = \frac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6\ \text{V}$$

Note that $V_1 + V_2 = 11.2$ V, which equals V_3 as it should.

Summary: $Q_1 = 22.4\ \mu\text{C},\ V_1 = 5.6$ V

$Q_2 = 22.4\ \mu\text{C},\ V_2 = 5.6$ V

$Q_3 = 44.8\ \mu\text{C},\ V_3 = 11.2$ V

$Q_4 = 67.2\ \mu\text{C},\ V_4 = 16.8$ V

(c) $V_{ad} = V_3 = 11.2$ V

EVALUATE: $V_1 + V_2 + V_4 = V$, or $V_3 + V_4 = V$. $Q_1 = Q_2$, $Q_1 + Q_3 = Q_4$ and $Q_4 = Q_{1234}$.

24.18. IDENTIFY: For capacitors in parallel the voltages are the same and the charges add. For capacitors in series, the charges are the same and the voltages add. $C = Q/V$.

SET UP: C_1 and C_2 are in parallel and C_3 is in series with the parallel combination of C_1 and C_2.

EXECUTE: **(a)** C_1 and C_2 are in parallel and so have the same potential across them:

$V_1 = V_2 = \dfrac{Q_2}{C_2} = \dfrac{40.0\times10^{-6}\ \text{C}}{3.00\times10^{-6}\ \text{F}} = 13.33\ \text{V}$. Therefore, $Q_1 = V_1C_1 = (13.33\ \text{V})(3.00\times10^{-6}\ \text{F}) = 80.0\times10^{-6}\ \text{C}$. Since C_3 is in series with the parallel combination of C_1 and C_2, its charge must be equal to their combined charge: $C_3 = 40.0\times10^{-6}\ \text{C} + 80.0\times10^{-6}\ \text{C} = 120.0\times10^{-6}\ \text{C}$.

(b) The total capacitance is found from $\dfrac{1}{C_{\text{tot}}} = \dfrac{1}{C_{12}} + \dfrac{1}{C_3} = \dfrac{1}{9.00\times10^{-6}\ \text{F}} + \dfrac{1}{5.00\times10^{-6}\ \text{F}}$ and $C_{\text{tot}} = 3.21\ \mu\text{F}$.

$$V_{ab} = \frac{Q_{\text{tot}}}{C_{\text{tot}}} = \frac{120.0\times10^{-6}\ \text{C}}{3.21\times10^{-6}\ \text{F}} = 37.4\ \text{V}.$$

EVALUATE: $V_3 = \dfrac{Q_3}{C_3} = \dfrac{120.0\times10^{-6}\ \text{C}}{5.00\times10^{-6}\ \text{F}} = 24.0\ \text{V}$. $V_{ab} = V_1 + V_3$.

24.22. IDENTIFY: Simplify the network by replacing series and parallel combinations of capacitors by their equivalents.

SET UP: For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$ For capacitors in parallel the voltages are the same and the charges add; $C_{\text{eq}} = C_1 + C_2 + \cdots$ $C = \dfrac{Q}{V}$.

EXECUTE: **(a)** The equivalent capacitance of the $5.0\ \mu\text{F}$ and $8.0\ \mu\text{F}$ capacitors in parallel is $13.0\ \mu\text{F}$. When these two capacitors are replaced by their equivalent we get the network sketched in Figure 24.22. The equivalent capacitance of these three capacitors in series is $3.47\ \mu\text{F}$.

(b) $Q_{\text{tot}} = C_{\text{tot}}V = (3.47\ \mu\text{F})(50.0\ \text{V}) = 174\ \mu\text{C}$

(c) Q_{tot} is the same as Q for each of the capacitors in the series combination shown in Figure 24.22, so Q for each of the capacitors is $174\ \mu\text{C}$.

EVALUATE: The voltages across each capacitor in Figure 24.22 are $V_{10} = \dfrac{Q_{\text{tot}}}{C_{10}} = 17.4\ \text{V}$, $V_{13} = \dfrac{Q_{\text{tot}}}{C_{13}} = 13.4\ \text{V}$ and $V_9 = \dfrac{Q_{\text{tot}}}{C_9} = 19.3\ \text{V}$. $V_{10} + V_{13} + V_9 = 17.4\ \text{V} + 13.4\ \text{V} + 19.3\ \text{V} = 50.1\ \text{V}$. The sum of the voltages equals the applied voltage, apart from a small difference due to rounding.

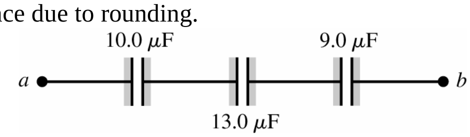


Figure 24.22

24.32. **IDENTIFY:** The two capacitors are in series. $\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}+\cdots$ $C=\frac{Q}{V}$. $U=\frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same.

EXECUTE: **(a)** $\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}$ so $C_{\text{eq}}=\frac{C_1C_2}{C_1+C_2}=\frac{(150\text{ nF})(120\text{ nF})}{150\text{ nF}+120\text{ nF}}=66.7\text{ nF}$.

$Q=CV=(66.7\text{ nF})(36\text{ V})=2.4\times10^{-6}\text{ C}=2.4\ \mu\text{C}$

(b) $Q=2.4\ \mu\text{C}$ for each capacitor.

(c) $U=\frac{1}{2}C_{\text{eq}}V^2=\frac{1}{2}(66.7\times10^{-9}\text{ F})(36\text{ V})^2=43.2\ \mu\text{J}$

(d) We know C and Q for each capacitor so rewrite U in terms of these quantities. $U=\frac{1}{2}CV^2=\frac{1}{2}C(Q/C)^2=Q^2/2C$

150 nF: $U=\frac{(2.4\times10^{-6}\text{ C})^2}{2(150\times10^{-9}\text{ F})}=19.2\ \mu\text{J}$; 120 nF: $U=\frac{(2.4\times10^{-6}\text{ C})^2}{2(120\times10^{-9}\text{ F})}=24.0\ \mu\text{J}$

Note that $19.2\ \mu\text{J}+24.0\ \mu\text{J}=43.2\ \mu\text{J}$, the total stored energy calculated in part (c).

(e) 150 nF: $V=\frac{Q}{C}=\frac{2.4\times10^{-6}\text{ C}}{150\times10^{-9}\text{ F}}=16\text{ V}$; 120 nF: $V=\frac{Q}{C}=\frac{2.4\times10^{-6}\text{ C}}{120\times10^{-9}\text{ F}}=20\text{ V}$

Note that these two voltages sum to 36 V, the voltage applied across the network.

EVALUATE: Since Q is the same the capacitor with smaller C stores more energy ($U=Q^2/2C$) and has a larger voltage ($V=Q/C$).

24.57. **IDENTIFY:** Simplify the network by replacing series and parallel combinations by their equivalent. The stored energy in a capacitor is $U=\frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same; $\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}+\cdots$. For capacitors in parallel the voltages are the same and the charges add; $C_{\text{eq}}=C_1+C_2+\cdots$ $C=\frac{Q}{V}$. $U=\frac{1}{2}CV^2$.

EXECUTE: **(a)** Find C_{eq} for the network by replacing each series or parallel combination by its equivalent. The successive simplified circuits are shown in Figure 24.57a–c.

$U_{\text{tot}}=\frac{1}{2}C_{\text{eq}}V^2=\frac{1}{2}(2.19\times10^{-6}\text{ F})(12.0\text{ V})^2=1.58\times10^{-4}\text{ J}=158\ \mu\text{J}$

(b) From Figure 24.57c, $Q_{\text{tot}}=C_{\text{eq}}V=(2.19\times10^{-6}\text{ F})(12.0\text{ V})=2.63\times10^{-5}\text{ C}$. From Figure 24.57b, $Q_{\text{tot}}=2.63\times10^{-5}\text{ C}$.

$V_{4.8}=\frac{Q_{4.8}}{C_{4.8}}=\frac{2.63\times10^{-5}\text{ C}}{4.80\times10^{-6}\text{ F}}=5.48\text{ V}$. $U_{4.8}=\frac{1}{2}CV^2=\frac{1}{2}(4.80\times10^{-6}\text{ F})(5.48\text{ V})^2=7.21\times10^{-5}\text{ J}=72.1\ \mu\text{J}$

This one capacitor stores nearly half the total stored energy.

EVALUATE: $U=\frac{Q^2}{2C}$. For capacitors in series the capacitor with the smallest C stores the greatest amount of energy.

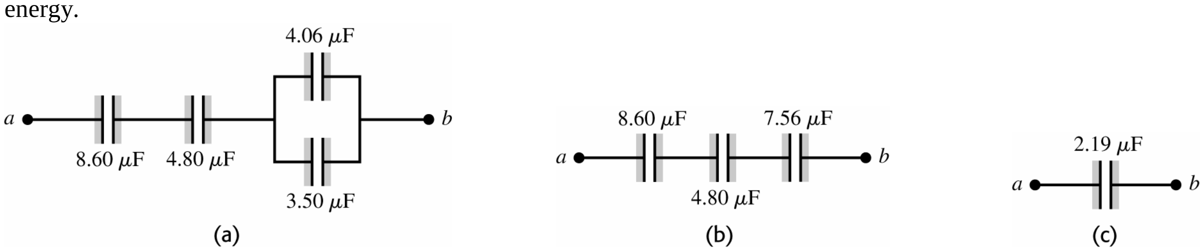
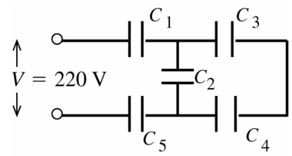


Figure 24.57

24.58. IDENTIFY: **Apply the rules for combining capacitors in series and parallel. For capacitors in series the voltages add**

24.59. **(a) IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents.

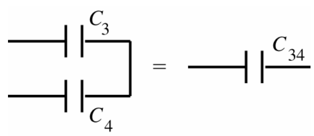
SET UP: The network is sketched in Figure 24.59a.



$C_1 = C_5 = 8.4\ \mu\text{F}$
$C_2 = C_3 = C_4 = 4.2\ \mu\text{F}$

Figure 24.59a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_3 and C_4 are in series and can be replaced by C_{34} (Figure 24.59b):

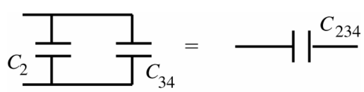


$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4}$$
$$\frac{1}{C_{34}} = \frac{C_3 + C_4}{C_3 C_4}$$

Figure 24.59b

$$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{(4.2\ \mu\text{F})(4.2\ \mu\text{F})}{4.2\ \mu\text{F} + 4.2\ \mu\text{F}} = 2.1\ \mu\text{F}$$

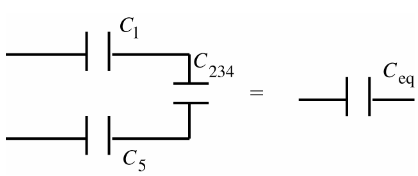
C_2 and C_{34} are in parallel and can be replaced by their equivalent (Figure 24.59c):



$$C_{234} = C_2 + C_{34}$$
$$C_{234} = 4.2\ \mu\text{F} + 2.1\ \mu\text{F}$$
$$C_{234} = 6.3\ \mu\text{F}$$

Figure 24.59c

C_1, C_5 and C_{234} are in series and can be replaced by C_{eq} (Figure 24.59d):



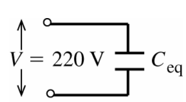
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_{234}}$$
$$\frac{1}{C_{\text{eq}}} = \frac{2}{8.4\ \mu\text{F}} + \frac{1}{6.3\ \mu\text{F}}$$
$$C_{\text{eq}} = 2.5\ \mu\text{F}$$

Figure 24.59d

EVALUATE: For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.

(b) IDENTIFY and **SET UP:** In each equivalent network apply the rules for *Q* and *V* for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

EXECUTE: The equivalent circuit is drawn in Figure 24.59e.



$$Q_{\text{eq}} = C_{\text{eq}} V$$
$$Q_{\text{eq}} = (2.5\ \mu\text{F})(220\ \text{V}) = 550\ \mu\text{C}$$

Figure 24.59e

$Q_1 = Q_5 = Q_{234} = 550\ \mu\text{C}$ (capacitors in series have same charge)

$$V_1 = \frac{Q_1}{C_1} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$

$$V_5 = \frac{Q_5}{C_5} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$

$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550\ \mu\text{C}}{6.3\ \mu\text{F}} = 87\ \text{V}$$

Now draw the network as in Figure 24.59f.

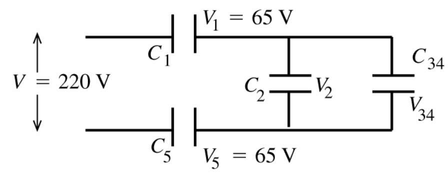


Figure 24.59f

$V_2 = V_{34} = V_{234} = 87$ V
capacitors in parallel have the same potential

$$Q_2 = C_2V_2 = (4.2\ \mu\text{F})(87\ \text{V}) = 370\ \mu\text{C}$$

$$Q_{34} = C_{34}V_{34} = (2.1\ \mu\text{F})(87\ \text{V}) = 180\ \mu\text{C}$$

Finally, consider the original circuit (Figure 24.59g).

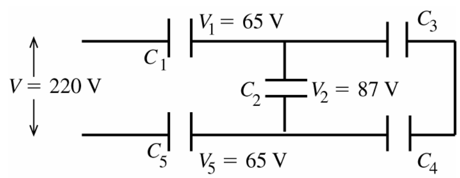


Figure 24.59g

$Q_3 = Q_4 = Q_{34} = 180\ \mu$C
capacitors in series have the same charge

$$V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}$$

$$V_4 = \frac{Q_4}{C_4} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}$$

Summary: $Q_1 = 550\ \mu\text{C},\ V_1 = 65\ \text{V}$

$Q_2 = 370\ \mu\text{C},\ V_2 = 87\ \text{V}$

$Q_3 = 180\ \mu\text{C},\ V_3 = 43\ \text{V}$

$Q_4 = 180\ \mu\text{C},\ V_4 = 43\ \text{V}$

$Q_5 = 550\ \mu\text{C},\ V_5 = 65\ \text{V}$

EVALUATE: $V_3 + V_4 = V_2$ and $V_1 + V_2 + V_5 = 220$ V (apart from some small rounding error)

$Q_1 = Q_2 + Q_3$ and $Q_5 = Q_2 + Q_4$