DIRECT-CURRENT CIRCUITS

26

26.1. **IDENTIFY:** The newly-formed wire is a combination of series and parallel resistors.
SET UP: Each of the three linear segments has resistance $R/3$. The circle is two $R/6$ resistors in parallel.
EXECUTE: The resistance of the circle is $R/12$ since it consists of two $R/6$ resistors in parallel. The equivalent resistance is two $R/3$ resistors in series with an $R/6$ resistor, giving $R_{\text{equiv}} = R/3 + R/3 + R/12 = 3R/4$.
EVALUATE: The equivalent resistance of the original wire has been reduced because the circle's resistance is less than it was as a linear wire.

26.4. **IDENTIFY:** For resistors in parallel the voltages are the same and equal to the voltage across the equivalent resistance.

SET UP: $V = IR$. $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$.

EXECUTE: **(a)** $R_{\text{eq}} = \left(\dfrac{1}{32\ \Omega} + \dfrac{1}{20\ \Omega}\right)^{-1} = 12.3\ \Omega$.

(b) $I = \dfrac{V}{R_{\text{eq}}} = \dfrac{240\text{ V}}{12.3\ \Omega} = 19.5\text{ A}$.

(c) $I_{32\Omega} = \dfrac{V}{R} = \dfrac{240\text{ V}}{32\ \Omega} = 7.5\text{ A}$; $I_{20\Omega} = \dfrac{V}{R} = \dfrac{240\text{ V}}{20\ \Omega} = 12\text{ A}$.

EVALUATE: More current flows through the resistor that has the smaller R.

26.6. **IDENTIFY:** The potential drop is the same across the resistors in parallel, and the current into the parallel combination is the same as the current through the 45.0-Ω resistor.
(a) SET UP: Apply Ohm's law in the parallel branch to find the current through the 45.0-Ω resistor. Then apply Ohm's law to the 45.0-Ω resistor to find the potential drop across it.
EXECUTE: The potential drop across the 25.0-Ω resistor is V_{25} = (25.0 Ω)(1.25 A) = 31.25 V. The potential drop across each of the parallel branches is 31.25 V. For the 15.0-Ω resistor: I_{15} = (31.25 V)/(15.0 Ω) = 2.083 A. The resistance of the 10.0-Ω + 15.0 Ω combination is 25.0 Ω, so the current through it must be the same as the current through the upper 25.0 Ω resistor: I_{10+15} = 1.25 A. The sum of currents in the parallel branch will be the current through the 45.0-Ω resistor.

$$I_{\text{Total}} = 1.25\text{ A} + 2.083\text{ A} + 1.25\text{ A} = 4.58\text{ A}$$

Apply Ohm's law to the 45.0 Ω resistor: V_{45} = (4.58 A)(45.0 Ω) = 206 V
(b) SET UP: First find the equivalent resistance of the circuit and then apply Ohm's law to it.
EXECUTE: The resistance of the parallel branch is $1/R$ = 1/(25.0 Ω) + 1/(15.0 Ω) + 1/(25.0 Ω), so R = 6.82 Ω. The equivalent resistance of the circuit is 6.82 Ω + 45.0 Ω + 35.00 Ω = 86.82 Ω. Ohm's law gives V_{Bat} = (86.62 Ω)(4.58 A) = 398 V.
EVALUATE: The emf of the battery is the sum of the potential drops across each of the three segments (parallel branch and two series resistors).

26.11. **IDENTIFY:** For resistors in parallel, the voltages are the same and the currents add. $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ so $R_{\text{eq}} = \dfrac{R_1 R_2}{R_1 + R_2}$,

For resistors in series, the currents are the same and the voltages add. $R_{\text{eq}} = R_1 + R_2$.

SET UP: The rules for combining resistors in series and parallel lead to the sequences of equivalent circuits shown in Figure 26.11.

EXECUTE: $R_{\text{eq}} = 5.00\ \Omega$. In Figure 26.11c, $I = \dfrac{60.0\text{ V}}{5.00\ \Omega} = 12.0\text{ A}$. This is the current through each of the resistors in Figure 26.11b. $V_{12} = IR_{12} = (12.0\text{ A})(2.00\ \Omega) = 24.0\text{ V}$. $V_{34} = IR_{34} = (12.0\text{ A})(3.00\ \Omega) = 36.0\text{ V}$. Note that $V_{12} + V_{34} = 60.0\text{ V}$. V_{12} is the voltage across R_1 and across R_2, so $I_1 = \frac{V_{12}}{R_1} = \frac{24.0\text{ V}}{3.00\ \Omega} = 8.00\text{ A}$ and $I_2 = \frac{V_{12}}{R_2} = \frac{24.0\text{ V}}{6.00\ \Omega} = 4.00\text{ A}$. V_{34} is the voltage across R_3 and across R_4, so $I_3 = \frac{V_{34}}{R_3} = \frac{36.0\text{ V}}{12.0\ \Omega} = 3.00\text{ A}$ and $I_4 = \frac{V_{34}}{R_4} = \frac{36.0\text{ V}}{4.00\ \Omega} = 9.00\text{ A}$.

EVALUATE: Note that $I_1 + I_2 = I_3 + I_4$.

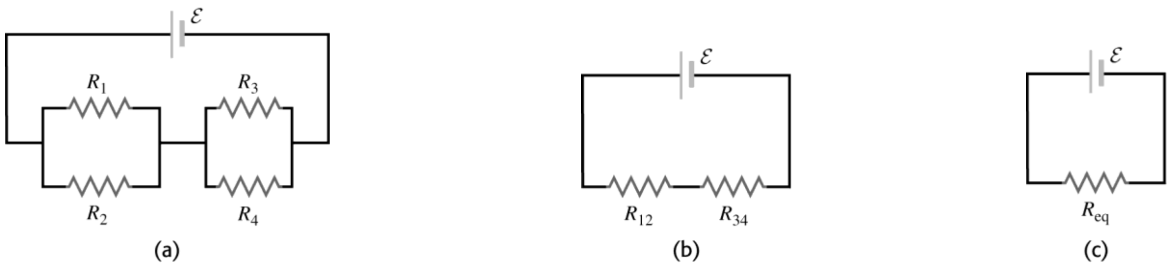

Figure 26.11

26.22. IDENTIFY: Apply the loop rule and junction rule.

SET UP: The circuit diagram is given in Figure 26.22. The junction rule has been used to find the magnitude and direction of the current in the middle branch of the circuit. There are no remaining unknown currents.

EXECUTE: The loop rule applied to loop (1) gives:

$$+20.0\text{ V} - (1.00\text{ A})(1.00\ \Omega) + (1.00\text{ A})(4.00\ \Omega) + (1.00\text{ A})(1.00\ \Omega) - \mathcal{E}_1 - (1.00\text{ A})(6.00\ \Omega) = 0$$

$\mathcal{E}_1 = 20.0\text{ V} - 1.00\text{ V} + 4.00\text{ V} + 1.00\text{ V} - 6.00\text{ V} = 18.0\text{ V}$. The loop rule applied to loop (2) gives:

$$+20.0\text{ V} - (1.00\text{ A})(1.00\ \Omega) - (2.00\text{ A})(1.00\ \Omega) - \mathcal{E}_2 - (2.00\text{ A})(2.00\ \Omega) - (1.00\text{ A})(6.00\ \Omega) = 0$$

$\mathcal{E}_2 = 20.0\text{ V} - 1.00\text{ V} - 2.00\text{ V} - 4.00\text{ V} - 6.00\text{ V} = 7.0\text{ V}$. Going from *b* to *a* along the lower branch, $V_b + (2.00\text{ A})(2.00\ \Omega) + 7.0\text{ V} + (2.00\text{ A})(1.00\ \Omega) = V_a$. $V_b - V_a = -13.0\text{ V}$; point *b* is at 13.0 V lower potential than point *a*.

EVALUATE: We can also calculate $V_b - V_a$ by going from *b* to *a* along the upper branch of the circuit. $V_b - (1.00\text{ A})(6.00\ \Omega) + 20.0\text{ V} - (1.00\text{ A})(1.00\ \Omega) = V_a$ and $V_b - V_a = -13.0\text{ V}$. This agrees with $V_b - V_a$ calculated along a different path between *b* and *a*.

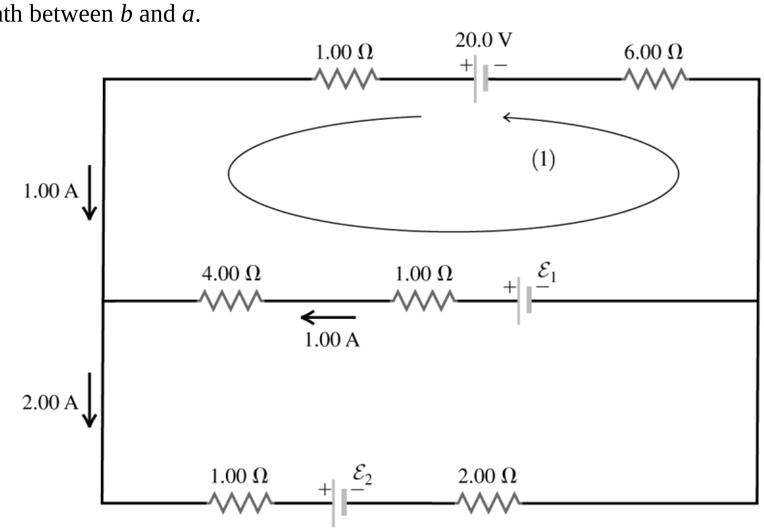

Figure 26.22

26.23. IDENTIFY: Apply the junction rule at points *a, b, c* and *d* to calculate the unknown currents. Then apply the loop rule to three loops to calculate $\mathcal{E}_1$, $\mathcal{E}_2$ and R.

(a) SET UP: The circuit is sketched in Figure 26.23.

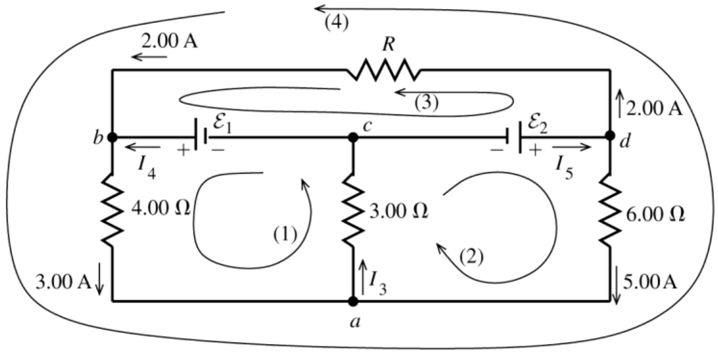

Figure 26.23

EXECUTE: Apply the junction rule to point *a*: $3.00\text{ A} + 5.00\text{ A} - I_3 = 0$

$I_3 = 8.00\text{ A}$

Apply the junction rule to point *b*: $2.00\text{ A} + I_4 - 3.00\text{ A} = 0$

$I_4 = 1.00\text{ A}$

Apply the junction rule to point *c*: $I_3 - I_4 - I_5 = 0$

$I_5 = I_3 - I_4 = 8.00\text{ A} - 1.00\text{ A} = 7.00\text{ A}$

EVALUATE: As a check, apply the junction rule to point *d*: $I_5 - 2.00\text{ A} - 5.00\text{ A} = 0$

$I_5 = 7.00\text{ A}$

(b) EXECUTE: Apply the loop rule to loop (1): $\mathcal{E}_1 - (3.00\text{ A})(4.00\ \Omega) - I_3(3.00\ \Omega) = 0$

$\mathcal{E}_1 = 12.0\text{ V} + (8.00\text{ A})(3.00\ \Omega) = 36.0\text{ V}$

Apply the loop rule to loop (2): $\mathcal{E}_2 - (5.00\text{ A})(6.00\ \Omega) - I_3(3.00\ \Omega) = 0$

$\mathcal{E}_2 = 30.0\text{ V} + (8.00\text{ A})(3.00\ \Omega) = 54.0\text{ V}$

(c) Apply the loop rule to loop (3): $-(2.00\text{ A})R - \mathcal{E}_1 + \mathcal{E}_2 = 0$

$$R = \frac{\mathcal{E}_2 - \mathcal{E}_1}{2.00\text{ A}} = \frac{54.0\text{ V} - 36.0\text{ V}}{2.00\text{ A}} = 9.00\ \Omega$$

EVALUATE: Apply the loop rule to loop (4) as a check of our calculations:

$-(2.00\text{ A})R - (3.00\text{ A})(4.00\ \Omega) + (5.00\text{ A})(6.00\ \Omega) = 0$

$-(2.00\text{ A})(9.00\ \Omega) - 12.0\text{ V} + 30.0\text{ V} = 0$

$-18.0\text{ V} + 18.0\text{ V} = 0$

26.27. **(a) IDENTIFY:** With the switch open, the circuit can be solved using series-parallel reduction.
SET UP: Find the current through the unknown battery using Ohm's law. Then use the equivalent resistance of the circuit to find the emf of the battery.
EXECUTE: The 30.0-Ω and 50.0-Ω resistors are in series, and hence have the same current. Using Ohm's law I_{50} = (15.0 V)/(50.0 Ω) = 0.300 A = I_{30}. The potential drop across the 75.0-Ω resistor is the same as the potential drop across the 80.0-Ω series combination. We can use this fact to find the current through the 75.0-Ω resistor using Ohm's law: $V_{75} = V_{80}$ = (0.300 A)(80.0 Ω) = 24.0 V and I_{75} = (24.0 V)/(75.0 Ω) = 0.320 A.
The current through the unknown battery is the sum of the two currents we just found:

$$I_{\text{Total}} = 0.300\text{ A} + 0.320\text{ A} = 0.620\text{ A}$$

The equivalent resistance of the resistors in parallel is $1/R_p$ = 1/(75.0 Ω) + 1/(80.0 Ω). This gives R_p = 38.7 Ω. The equivalent resistance "seen" by the battery is R_{equiv} = 20.0 Ω + 38.7 Ω = 58.7 Ω.
Applying Ohm's law to the battery gives $\mathcal{E} = R_{\text{equiv}} I_{\text{Total}}$ = (58.7 Ω)(0.620 A) = 36.4 V
(b) IDENTIFY: With the switch closed, the 25.0-V battery is connected across the 50.0-Ω resistor.
SET UP: Taking a loop around the right part of the circuit.
EXECUTE: Ohm's law gives I = (25.0 V)/(50.0 Ω) = 0.500 A
EVALUATE: The current through the 50.0-Ω resistor, and the rest of the circuit, depends on whether or not the switch is open.

26.39. **IDENTIFY:** The capacitor discharges exponentially through the voltmeter. Since the potential difference across the capacitor is directly proportional to the charge on the plates, the voltage across the plates decreases exponentially with the same time constant as the charge.

SET UP: The reading of the voltmeter obeys the equation $V = V_0 e^{-t/RC}$, where RC is the time constant.
EXECUTE: **(a)** Solving for C and evaluating the result when t = 4.00 s gives

$$C = \frac{t}{R\ln V/V_0} = \frac{4.00\text{ s}}{(3.40\times 10^{6}\ \Omega)\ln\left(\dfrac{12.0\text{ V}}{3.00\text{ V}}\right)} = 8.49\times 10^{-7}\text{ F}$$

(b) $\tau = RC = (3.40\times 10^{6}\ \Omega)(8.49\times 10^{-7}\text{ F}) = 2.89$ s
EVALUATE: In most laboratory circuits, time constants are much shorter than this one.

26.46. **IDENTIFY:** Both the charge and energy decay exponentially, but not with the same time constant since the energy is proportional to the *square* of the charge.
SET UP: The charge obeys the equation $Q = Q_0\, e^{-t/RC}$ but the energy obeys the equation $U = Q^2/2C = (Q_0\, e^{-t/RC})/2C = U_0\, e^{-2t/RC}$.
EXECUTE: **(a)** The charge is reduced by half: $Q_0/2 = Q_0\, e^{-t/RC}$. This gives

$$t = RC\ln 2 = (175\ \Omega)(12.0\ \mu\text{F})(\ln 2) = 1.456\text{ ms} = 1.46\text{ ms}.$$

(b) The energy is reduced by half: $U_0/2 = U_0\, e^{-2t/RC}$. This gives

$$t = (RC\ln 2)/2 = (1.456\text{ ms})/2 = 0.728\text{ ms}.$$

EVALUATE: The energy decreases faster than the charge because it is proportional to the square of the charge.

26.61. **IDENTIFY:** Apply the junction rule to express the currents through the $5.00\ \Omega$ and $8.00\ \Omega$ resistors in terms of I_1, I_2 and I_3. Apply the loop rule to three loops to get three equations in the three unknown currents.
SET UP: The circuit is sketched in Figure 26.61.

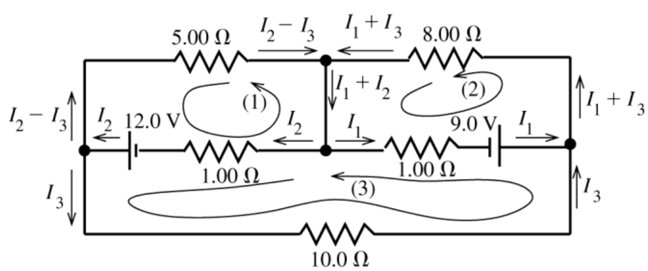

Figure 26.61

The current in each branch has been written in terms of I_1, I_2 and I_3 such that the junction rule is satisfied at each junction point.
EXECUTE: Apply the loop rule to loop (1).

$-12.0\text{ V} + I_2\ \ 1.00\ \Omega\ +\ I_2 - I_3\ \ 5.00\ \Omega\ = 0$

$I_2\ \ 6.00\ \Omega\ - I_3\ \ 5.00\ \Omega\ = 12.0\text{ V}$ eq.(1)

Apply the loop rule to loop (2).

$-I_1\ \ 1.00\ \Omega\ + 9.00\text{ V} -\ I_1 + I_3\ \ 8.00\ \Omega\ = 0$

$I_1\ \ 9.00\ \Omega\ + I_3\ \ 8.00\ \Omega\ = 9.00\text{ V}$ eq.(2)

Apply the loop rule to loop (3).

$-I_3\ \ 10.0\ \Omega\ - 9.00\text{ V} + I_1\ \ 1.00\ \Omega\ - I_2\ \ 1.00\ \Omega\ + 12.0\text{ V} = 0$

$-I_1\ \ 1.00\ \Omega\ + I_2\ \ 1.00\ \Omega\ + I_3\ \ 10.0\ \Omega\ = 3.00\text{ V}$ eq.(3)

Eq.(1) gives $I_2 = 2.00\text{ A} + \frac{5}{6}I_3$; eq.(2) gives $I_1 = 1.00\text{ A} - \frac{8}{9}I_3$

Using these results in eq.(3) gives $-\ 1.00\text{ A} - \frac{8}{9}I_3\ \ 1.00\ \Omega\ +\ 2.00\text{ A} + \frac{5}{6}I_3\ \ 1.00\ \Omega\ + I_3\ \ 10.0\ \Omega\ = 3.00\text{ V}$

$\frac{16+15+180}{18}\ \ I_3 = 2.00\text{ A};\ I_3 = \frac{18}{211}\ \ 2.00\text{ A}\ = 0.171\text{ A}$

Then $I_2 = 2.00\text{ A} + \frac{5}{6}I_3 = 2.00\text{ A} + \frac{5}{6}\ \ 0.171\text{ A}\ = 2.14\text{ A}$ and $I_1 = 1.00\text{ A} - \frac{8}{9}I_3 = 1.00\text{ A} - \frac{8}{9}\ \ 0.171\text{ A}\ = 0.848\text{ A}$.

EVALUATE: We could check that the loop rule is satisfied for a loop that goes through the $5.00\ \Omega$, $8.00\ \Omega$ and $10.0\ \Omega$ resistors. Going around the loop clockwise: $-\ I_2 - I_3\ \ 5.00\ \Omega\ +\ I_1 + I_3\ \ 8.00\ \Omega\ + I_3\ \ 10.0\ \Omega\ = -9.85\text{ V} + 8.15\text{ V} + 1.71\text{ V}$, which does equal zero, apart from rounding.

Figure 26.93

26.85. **IDENTIFY:** $q = Q_0 e^{-t/RC}$. The time constant is $\tau = RC$.

SET UP: The charge of one electron has magnitude $e = 1.60 \times 10^{-19}$ C.

EXECUTE: **(a)** We will say that a capacitor is discharged if its charge is less than that of one electron. The time this takes is then given by $q = Q_0 e^{-t/RC}$, so $t = RC\ln(Q_0/e) = (6.7\times10^5\ \Omega)(9.2\times10^{-7}\ \text{F})\ln(7.0\times10^{-6}\ \text{C}/1.6\times10^{-19}\ \text{C}) = 19.36$ s, or 31.4 time constants.

EVALUATE: **(b)** As shown in part (a), $t = \tau\ln(Q_0/q)$ and so the number of time constants required to discharge the capacitor is independent of R and C, and depends only on the initial charge.