27

MAGNETIC FIELD AND MAGNETIC FORCES

27.1. **IDENTIFY** and **SET UP:** Apply Eq.(27.2) to calculate $\vec{F}$. Use the cross products of unit vectors from Section 1.10.

EXECUTE: $\vec{v} = (+4.19\times10^{4}\ \text{m/s})\hat{i} + (-3.85\times10^{4}\ \text{m/s})\hat{j}$

(a) $\vec{B} = (1.40\ \text{T})\hat{i}$

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8}\ \text{C})(1.40\ \text{T})\left[(4.19\times10^{4}\ \text{m/s})\hat{i}\times\hat{i} - (3.85\times10^{4}\ \text{m/s})\hat{j}\times\hat{i}\right]$

$\hat{i}\times\hat{i} = 0,\ \hat{j}\times\hat{i} = -\hat{k}$

$\vec{F} = (-1.24\times10^{-8}\ \text{C})(1.40\ \text{T})(-3.85\times10^{4}\ \text{m/s})(-\hat{k}) = (-6.68\times10^{-4}\ \text{N})\hat{k}$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1a.

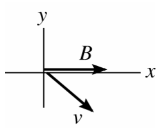


Figure 27.1a

The right-hand rule gives that $\vec{v}\times\vec{B}$ is directed out of the paper (+z-direction). The charge is negative so $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$;

$\vec{F}$ is in the $-z$-direction. This agrees with the direction calculated with unit vectors.

(b) EXECUTE: $\vec{B} = (1.40\ \text{T})\hat{k}$

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8}\ \text{C})(1.40\ \text{T})\left[(+4.19\times10^{4}\ \text{m/s})\hat{i}\times\hat{k} - (3.85\times10^{4}\ \text{m/s})\hat{j}\times\hat{k}\right]$

$\hat{i}\times\hat{k} = -\hat{j},\ \hat{j}\times\hat{k} = \hat{i}$

$\vec{F} = (-7.27\times10^{-4}\ \text{N})(-\hat{j}) + (6.68\times10^{-4}\ \text{N})\hat{i} = \left[(6.68\times10^{-4}\ \text{N})\hat{i} + (7.27\times10^{-4}\ \text{N})\hat{j}\right]$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1b.

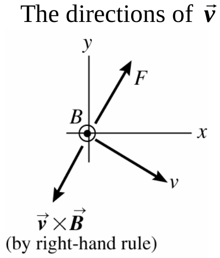


Figure 27.1b

The direction of $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$ since q is negative. The direction of $\vec{F}$ computed from the right-hand rule agrees qualitatively with the direction calculated with unit vectors.

27.5. **IDENTIFY:** Apply $F = |q|vB\sin\phi$ and solve for v.

SET UP: An electron has $q = -1.60\times10^{-19}\ \text{C}$.

EXECUTE: $v = \dfrac{F}{|q|B\sin\phi} = \dfrac{4.60\times10^{-15}\ \text{N}}{(1.6\times10^{-19}\ \text{C})(3.5\times10^{-3}\ \text{T})\sin 60^\circ} = 9.49\times10^{6}\ \text{m/s}$

EVALUATE: Only the component $B\sin\phi$ of the magnetic field perpendicular to the velocity contributes to the force.

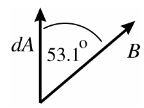
27.11. **IDENTIFY** and **SET UP:** $\Phi_B = \int \vec{B}\cdot d\vec{A}$

Circular area in the *xy*-plane, so $A = \pi r^2 = \pi(0.0650\text{ m})^2 = 0.01327\text{ m}^2$ and $d\vec{A}$ is in the *z*-direction. Use Eq.(1.18) to calculate the scalar product.

EXECUTE: (a) $\vec{B} = (0.230\text{ T})\hat{k}$; $\vec{B}$ and $d\vec{A}$ are parallel $(\phi = 0°)$ so $\vec{B}\cdot d\vec{A} = B\,dA.$

B is constant over the circular area so $\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\,dA = B\int dA = BA = (0.230\text{ T})(0.01327\text{ m}^2) = 3.05\times10^{-3}\text{ Wb}$

(b) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11a.



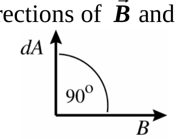
$\vec{B}\cdot d\vec{A} = B\cos\phi\, dA$
with $\phi = 53.1°$

Figure 27.11a

B and ϕ are constant over the circular area so $\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\cos\phi\, dA = B\cos\phi \int dA = B\cos\phi A$

$\Phi_B = (0.230\text{ T})\cos 53.1°(0.01327\text{ m}^2) = 1.83\times10^{-3}\text{ Wb}$

(c) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11b.



$\vec{B}\cdot d\vec{A} = 0$ since $d\vec{A}$ and $\vec{B}$ are perpendicular $(\phi = 90°)$

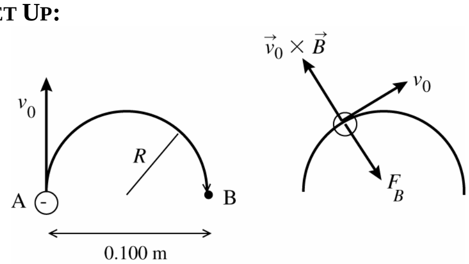
$\Phi_B = \int \vec{B}\cdot d\vec{A} = 0.$

Figure 27.11b

EVALUATE: Magnetic flux is a measure of how many magnetic field lines pass through the surface. It is maximum when $\vec{B}$ is perpendicular to the plane of the loop (part a) and is zero when $\vec{B}$ is parallel to the plane of the loop (part c).

27.15. **(a) IDENTIFY:** Apply Eq.(27.2) to relate the magnetic force $\vec{F}$ to the directions of $\vec{v}$ and $\vec{B}$. The electron has negative charge so $\vec{F}$ is opposite to the direction of $\vec{v}\times\vec{B}$. For motion in an arc of a circle the acceleration is toward the center of the arc so $\vec{F}$ must be in this direction. $a = v^2/R.$

SET UP:



As the electron moves in the semicircle, its velocity is tangent to the circular path. The direction of $\vec{v}_0\times\vec{B}$ at a point along the path is shown in Figure 27.15.

Figure 27.15

EXECUTE: For circular motion the acceleration of the electron $\vec{a}_{\text{rad}}$ is directed in toward the center of the circle. Thus the force $\vec{F}_B$ exerted by the magnetic field, since it is the only force on the electron, must be radially inward. Since q is negative, $\vec{F}_B$ is opposite to the direction given by the right-hand rule for $\vec{v}_0\times\vec{B}$. Thus $\vec{B}$ is directed into the page. Apply Newton's 2nd law to calculate the magnitude of $\vec{B}$: $\sum\vec{F} = m\vec{a}$ gives $\sum F_{\text{rad}} = ma$

$F_B = m(v^2/R)$

$F_B = |q|vB\sin\phi = |q|vB,$ so $|q|vB = m(v^2/R)$

$$B = \frac{mv}{|q|R} = \frac{(9.109\times10^{-31}\text{ kg})(1.41\times10^6\text{ m/s})}{(1.602\times10^{-19}\text{ C})(0.050\text{ m})} = 1.60\times10^{-4}\text{ T}$$

(b) IDENTIFY and **SET UP:** The speed of the electron as it moves along the path is constant. ($\vec{F}_B$ changes the direction of $\vec{v}$ but not its magnitude.) The time is given by the distance divided by v_0.

EXECUTE: The distance along the semicircular path is πR, so $t = \dfrac{\pi R}{v_0} = \dfrac{\pi(0.050\text{ m})}{1.41\times10^6\text{ m/s}} = 1.11\times10^{-7}\text{ s}$

EVALUATE: The magnetic field required increases when v increases or R decreases and also depends on the mass to charge ratio of the particle.

27.30. **IDENTIFY:** For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection.

EXECUTE: To pass undeflected in both cases, $E = vB = (5.85\times10^3\ \text{m/s})(1.35\ \text{T}) = 7898\ \text{N/C}$.

(a) If $q = 0.640\times10^{-9}$ C, the electric field direction is given by $-(\hat{\boldsymbol{j}}\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since it must point in the opposite direction to the magnetic force.

(b) If $q = -0.320\times10^{-9}$ C, the electric field direction is given by $((-\hat{\boldsymbol{j}})\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since the electric force must point in the opposite direction as the magnetic force. Since the particle has negative charge, the electric force is opposite to the direction of the electric field and the magnetic force is opposite to the direction it has in part (a).

EVALUATE: The same configuration of electric and magnetic fields works as a velocity selector for both positively and negatively charged particles.

27.35. **IDENTIFY:** Apply $F = IlB\sin\phi$.

SET UP: Label the three segments in the field as *a*, *b*, and *c*. Let *x* be the length of segment *a*. Segment *b* has length 0.300 m and segment *c* has length $0.600\ \text{cm} - x$. Figure 27.35a shows the direction of the force on each segment. For each segment, $\phi = 90°$. The total force on the wire is the vector sum of the forces on each segment.

EXECUTE: $F_a = IlB = (4.50\ \text{A})x(0.240\ \text{T})$. $F_c = (4.50\ \text{A})(0.600\ \text{m} - x)(0.240\ \text{T})$. Since $\vec{\boldsymbol{F}}_a$ and $\vec{\boldsymbol{F}}_c$ are in the same direction their vector sum has magnitude $F_{ac} = F_a + F_c = (4.50\ \text{A})(0.600\ \text{m})(0.240\ \text{T}) = 0.648\ \text{N}$ and is directed toward the bottom of the page in Figure 27.35a. $F_b = (4.50\ \text{A})(0.300\ \text{m})(0.240\ \text{T}) = 0.324\ \text{N}$ and is directed to the right. The vector addition diagram for $\vec{\boldsymbol{F}}_{ac}$ and $\vec{\boldsymbol{F}}_b$ is given in Figure 27.35b.

$F = \sqrt{F_{ac}^2 + F_b^2} = \sqrt{(0.648\ \text{N})^2 + (0.324\ \text{N})^2} = 0.724\ \text{N}$. $\tan\theta = \dfrac{F_{ac}}{F_b} = \dfrac{0.648\ \text{N}}{0.324\ \text{N}}$ and $\theta = 63.4°$. The net force has magnitude 0.724 N and its direction is specified by $\theta = 63.4°$ in Figure 27.35b.

EVALUATE: All three current segments are perpendicular to the magnetic field, so $\phi = 90°$ for each in the force equation. The direction of the force on a segment depends on the direction of the current for that segment.

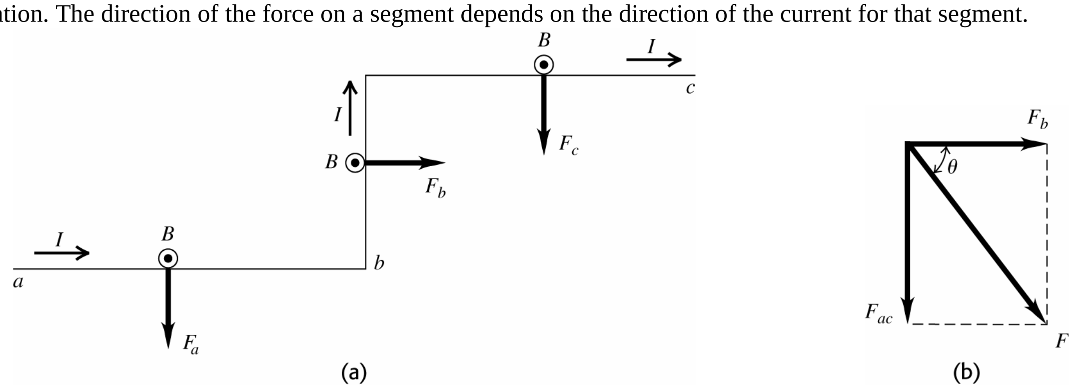


Figure 27.35

27.42. **IDENTIFY:** $\tau = IAB\sin\phi$. The magnetic moment of the loop is $\mu = IA$.

SET UP: Since the plane of the loop is parallel to the field, the field is perpendicular to the normal to the loop and $\phi = 90°$.

EXECUTE: **(a)** $\tau = IAB = (6.2\ \text{A})(0.050\ \text{m})(0.080\ \text{m})(0.19\ \text{T}) = 4.7\times10^{-3}\ \text{N}\cdot\text{m}$

(b) $\mu = IA = (6.2\ \text{A})(0.050\ \text{m})(0.080\ \text{m}) = 0.025\ \text{A}\cdot\text{m}^2$

EVALUATE: The torque is a maximum when the field is in the plane of the loop and $\phi = 90°$.

27.44. **IDENTIFY:** $\tau = IAB\sin\phi$, where ϕ is the angle between $\vec{\boldsymbol{B}}$ and the normal to the loop.

SET UP: The coil as viewed along the axis of rotation is shown in Figure 27.44a for its original position and in Figure 27.44b after it has rotated $30.0°$.

EXECUTE: **(a)** The forces on each side of the coil are shown in Figure 27.44a. $\vec{\boldsymbol{F}}_1 + \vec{\boldsymbol{F}}_2 = 0$ and $\vec{\boldsymbol{F}}_3 + \vec{\boldsymbol{F}}_4 = 0$. The net force on the coil is zero. $\phi = 0°$ and $\sin\phi = 0$, so $\tau = 0$. The forces on the coil produce no torque.

(b) The net force is still zero. $\phi = 30.0°$ and the net torque is

$\tau = (1)(1.40\text{ A})(0.220\text{ m})(0.350\text{ m})(1.50\text{ T})\sin 30.0° = 0.0808\text{ N}\cdot\text{m}$. The net torque is clockwise in Figure 27.44b and is directed so as to increase the angle ϕ .

EVALUATE: For any current loop in a uniform magnetic field the net force on the loop is zero. The torque on the loop depends on the orientation of the plane of the loop relative to the magnetic field direction.

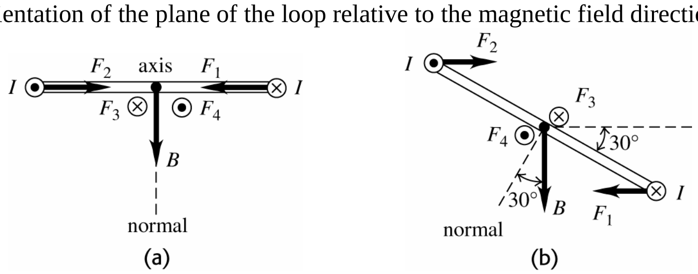


Figure 27.44