28

SOURCES OF MAGNETIC FIELD

28.3. **IDENTIFY:** A moving charge creates a magnetic field.

SET UP: The magnetic field due to a moving charge is $B=\dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$.

EXECUTE: Substituting numbers into the above equation gives

(a) $B=\dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}=\dfrac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\dfrac{(1.6\times10^{-19}\text{ C})(3.0\times10^{7}\text{ m/s})\sin30°}{(2.00\times10^{-6}\text{ m})^2}$.

$B = 6.00 \times 10^{-8}$ T, out of the paper, and it is the same at point B.

(b) $B = (1.00 \times 10^{-7}\text{ T}\cdot\text{m/A})(1.60 \times 10^{-19}\text{ C})(3.00 \times 10^{7}\text{ m/s})/(2.00 \times 10^{-6}\text{ m})^2$

$B = 1.20 \times 10^{-7}$ T, out of the page.

(c) $B = 0$ T since $\sin(180°) = 0$.

EVALUATE: Even at high speeds, these charges produce magnetic fields much less than the Earth's magnetic field.

28.16. **IDENTIFY:** The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B=\dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: First find the current: $I = (3.50 \times 10^{18}\text{ el/s})(1.60 \times 10^{-19}\text{ C/el}) = 0.560$ A

Now find the magnetic field: $\dfrac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(0.560\text{ A})}{2\pi(0.0400\text{ m})} = 2.80 \times 10^{-6}$ T

Since electrons are negative, the conventional current runs from east to west, so the magnetic field above the wire points toward the north.

EVALUATE: This magnetic field is much less than that of the Earth, so any experiments involving such a current would have to be shielded from the Earth's magnetic field, or at least would have to take it into consideration.

28.21. **IDENTIFY:** $B=\dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule in Section 20.7.

SET UP: Call the wires a and b, as indicated in Figure 28.21. The magnetic fields of each wire at points P_1 and P_2 are shown in Figure 28.21a. The fields at point 3 are shown in Figure 28.21b.

EXECUTE: **(a)** At P_1, $B_a=B_b$ and the two fields are in opposite directions, so the net field is zero.

(b) $B_a=\dfrac{\mu_0 I}{2\pi r_a}$. $B_b=\dfrac{\mu_0 I}{2\pi r_b}$. $\vec{B}_a$ and $\vec{B}_b$ are in the same direction so

$$B=B_a+B_b=\frac{\mu_0 I}{2\pi}\left(\frac{1}{r_a}+\frac{1}{r_b}\right)=\frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(4.00\text{ A})}{2\pi}\left[\frac{1}{0.300\text{ m}}+\frac{1}{0.200\text{ m}}\right]=6.67\times10^{-6}\text{ T}$$

$\vec{B}$ has magnitude $6.67\ \mu$T and is directed toward the top of the page.

(c) In Figure 28.21b, $\vec{B}_a$ is perpendicular to $\vec{r}_a$ and $\vec{B}_b$ is perpendicular to $\vec{r}_b$. $\tan\theta=\dfrac{5\text{ cm}}{20\text{ cm}}$ and $\theta=14.04°$.

$r_a=r_b=\sqrt{(0.200\text{ m})^2+(0.050\text{ m})^2}=0.206$ m and $B_a=B_b$.

$$B=B_a\cos\theta+B_b\cos\theta=2B_a\cos\theta=2\left(\frac{\mu_0 I}{2\pi r_a}\right)\cos\theta=\frac{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(4.0\text{ A})\cos14.04°}{2\pi(0.206\text{ m})}=7.54\ \mu\text{T}$$

B has magnitude $7.53\ \mu$T and is directed to the left.

EVALUATE: At points directly to the left of both wires the net field is directed toward the bottom of the page.

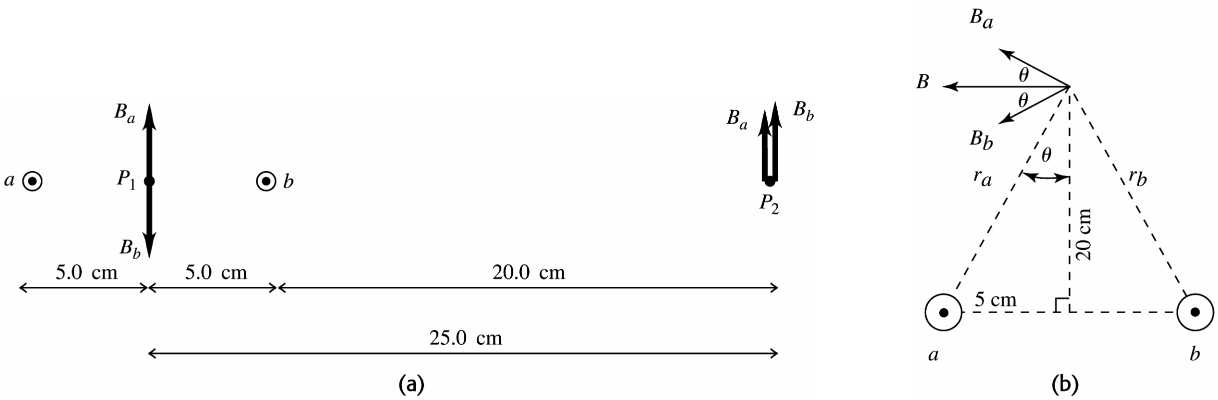

Figure 28.21

28.23. **IDENTIFY:** The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

SET UP: For each wire, $B = \dfrac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook.

EXECUTE: **(a)** and **(b)** $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.

(c) The fields due to each wire are sketched in Figure 28.23.

$$B = B_a \cos 45° + B_b \cos 45° + B_c \cos 45° + B_d \cos 45° = 4B_a \cos 45° = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45° .$$

$r = \sqrt{(10\text{ cm})^2 + (10\text{ cm})^2} = 10\sqrt{2}\text{ cm} = 0.10\sqrt{2}\text{ m}$, so

$$B = 4\frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(100\text{ A})}{2\pi(0.10\sqrt{2}\text{ m})}\cos 45° = 4.0\times 10^{-4}\text{ T, to the left.}$$

EVALUATE: In part (c), if all four currents are reversed in direction, the net field at the center of the square would be to the right.

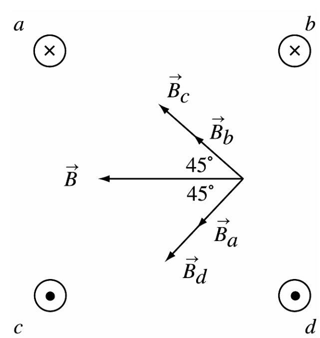

Figure 28.23

28.25. **IDENTIFY:** Apply Eq.(28.11).

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: **(a)** $F = \dfrac{\mu_0 I_1 I_2 L}{2\pi r} = \dfrac{\mu_0 (5.00\text{ A})(2.00\text{ A})(1.20\text{ m})}{2\pi(0.400\text{ m})} = 6.00\times 10^{-6}\text{ N}$, and the force is repulsive since the currents are in opposite directions.

(b) Doubling the currents makes the force increase by a factor of four to $F = 2.40\times 10^{-5}\text{ N}$.

EVALUATE: Doubling the current in a wire doubles the magnetic field of that wire. For fixed magnetic field, doubling the current in a wire doubles the force that the magnetic field exerts on the wire.

28.30. **IDENTIFY:** The magnetic field at the center of a circular loop is $B = \dfrac{\mu_0 I}{2R}$. By symmetry each segment of the loop that has length Δl contributes equally to the field, so the field at the center of a semicircle is $\frac{1}{2}$ that of a full loop.

SET UP: Since the straight sections produce no field at *P*, the field at *P* is $B = \dfrac{\mu_0 I}{4R}$.

EXECUTE: $B = \dfrac{\mu_0 I}{4R}$. The direction of $\vec{B}$ is given by the right-hand rule: $\vec{B}$ is directed into the page.

EVALUATE: For a quarter-circle section of wire the magnetic field at its center of curvature is $B = \dfrac{\mu_0 I}{8R}$.

28.35. **IDENTIFY:** Apply Ampere's law.
SET UP: $\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$

EXECUTE: **(a)** $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} = 3.83\times 10^{-4}\ \text{T}\cdot\text{m}$ and $I_{\text{encl}} = 305$ A.

(b) $-3.83\times 10^{-4}\ \text{T}\cdot\text{m}$ since at each point on the curve the direction of $d\vec{l}$ is reversed.

EVALUATE: The line integral $\oint \vec{B}\cdot d\vec{l}$ around a closed path is proportional to the net current that is enclosed by the path.

28.63. **IDENTIFY:** Apply $\sum \vec{F} = \mathbf{0}$ to one of the wires. The force one wire exerts on the other depends on I so $\sum \vec{F} = \mathbf{0}$ gives two equations for the two unknowns T and I.
SET UP: The force diagram for one of the wires is given in Figure 28.63.

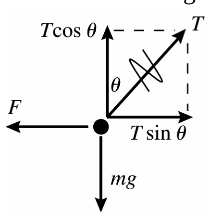

Figure 28.63

The force one wire exerts on the other is $F = \left(\dfrac{\mu_0 I^2}{2\pi r}\right)L$, where $r = 2(0.040\ \text{m})\sin\theta = 8.362\times 10^{-3}$ m is the distance between the two wires.

EXECUTE: $\sum F_y = 0$ gives $T\cos\theta = mg$ and $T = mg/\cos\theta$

$\sum F_x = 0$ gives $F = T\sin\theta = (mg/\cos\theta)\sin\theta = mg\tan\theta$

And $m = \lambda L$, so $F = \lambda L g \tan\theta$

$$\left(\frac{\mu_0 I^2}{2\pi r}\right)L = \lambda L g \tan\theta$$

$$I = \sqrt{\frac{\lambda g r \tan\theta}{(\mu_0/2\pi)}}$$

$$I = \sqrt{\frac{(0.0125\ \text{kg/m})(9.80\ \text{m/s})^2(\tan\ 6.00°)(8.362\times 10^{-3}\ \text{m})}{2\times 10^{-7}\,\text{T}\cdot\text{m/A}}} = 23.2\ \text{A}$$

EVALUATE: Since the currents are in opposite directions the wires repel. When I is increased, the angle θ from the vertical increases; a large current is required even for the small displacement specified in this problem.

28.64. **IDENTIFY:** Consider the forces on each side of the loop.
SET UP: The forces on the left and right sides cancel. The forces on the top and bottom segments of the loop are in opposite directions, so the magnitudes subtract.

EXECUTE: $F = F_{\text{t}} - F_{\text{b}} = \left(\dfrac{\mu_0 I_{\text{wire}}}{2\pi}\right)\left(\dfrac{Il}{r_{\text{t}}} - \dfrac{Il}{r_{\text{b}}}\right) = \dfrac{\mu_0 I l I_{\text{wire}}}{2\pi}\left(\dfrac{1}{r_{\text{r}}} - \dfrac{1}{r_{\text{l}}}\right)$.

$F = \dfrac{\mu_0(5.00\ \text{A})(0.200\ \text{m})(14.0\ \text{A})}{2\pi}\left(\dfrac{1}{0.100\ \text{m}} - \dfrac{1}{0.026\ \text{m}}\right) = 7.97\times 10^{-5}$ N. The force on the top segment is away from the wire, so the net force is away from the wire.

EVALUATE: The net force on a current loop in a uniform magnetic field is zero, but the magnetic field of the wire is not uniform, it is stronger closer to the wire.